INSTRUCTIONAL APPROACHES IN THE TEACHING OF EUCLIDEAN GEOMETRY IN GRADE 11

By

S.G. MTHEMBU

A Dissertation submitted to the Faculty of Education

Of

University of KwaZulu-Natal

In partial fulfillment of the requirement for the

Degree of Master of Education
(Mathematics Education)

Supervisor: Dr. D. Brijlall

March 2007

PREFACE

The research project described in this dissertation was carried out in three schools falling under Lower Umvoti Ward, in the Maphumulo circuit of KwaZulu Natal. The project commenced from June 2005 to May 2006 under the supervision of Dr D. Brijlall of the University of KwaZulu Natal, Durban.

This study represents the original work by the author and has not been submitted in any form for any degree or diploma to any tertiary institution. Where the author has made use of work of others, it is duly acknowledged in the text.

..... Date: 12-12-2007

Sibusiso Goodenough Mthembu

Student Number: 911 353 881

..... Date: 12-12-2007

Dr. D. Brijlall

Supervisor

Durban

ACKNOWLEDGEMENTS

First of all I would like to thank God for keeping me alive and helping me through this dissertation especially during the times when I tended to loose hope and thought of giving up.

Among the many people who helped me in completing this dissertation I particularly want to thank Dr. D. Brijlall for his continuous encouragement and the role he played as my supervisor. I would also like to extend my sincere appreciation to educators and learners who participated in the study. The experiences of educators in their teaching and the knowledge they have for mathematics widened my understanding of teaching conditions in schools.

My sincere gratitude also goes to my wife Thembi who has always been besides me, encouraging me to continue with the project. Finally special thanks goes to the rest of my family members, my two daughters Silondiwe and Nonhlanhla, two sons Siphiwe and Nala as well as my mother Buyisiwe for their continuous support.

ABSTRACT

The main focus of the research was to find out the causes of a poor performance in euclidean geometry especially in a grade eleven class. An easier way to find that information was to investigate the techniques that educators who are teaching grade eleven are following when they teach euclidean geometry. The necessary data was therefore collected from the educators as well as learners who were in grade eleven.

This study is guided by the constructivist's view. The theoretical framework of this research is based on the ideas of theorists like Piaget, Vygotsky and other authors who conform to constructivism. Changes that affected the education system of South Africa due to the adoption of the new constitution were also visited. A shift from the traditional way of teaching and an Outcomes Based Education system, as a recommendation by the National Curriculum Statement was highlighted.

The data was collected through both interviews and questionnaires. The semi–structured interviews of three educators from three participating schools were audio taped. In each school one educator was interviewed and six learners were given questionnaires to answer.

The above gave a total of eighteen learners and three educators. Written responses from learners and audio taped responses from educators were kept and analyzed. The interview was focused on the techniques that educators employ in their teaching of euclidean geometry in grade eleven. The questionnaires administered to learners were aimed at confirming the responses from the educators.

It is envisaged that the educators participated in the study can provide enough information which can assist in correcting the teaching approach in euclidean geometry. The findings show that the conditions under which educators teach contribute to their methods of teaching euclidean geometry. The testing system and the focus on better results by the education department proved to be the main determining factors of the methods that educators resort to when they teach learners.

It also came up from this study that some learners do not take mathematics out of their will. Their parents or the school forces them to take mathematics. Those who like to take mathematics are constantly discouraged by comments of educators who deem mathematics as a subject responsible for bringing down the pass rate of the school.

The above diminishes the love of mathematics to learners and euclidean geometry becomes the section that suffers the most. Suggestions and recommendations aimed at improving the teaching and learning of the euclidean geometry have been made.

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CHAPTER 1

INTRODUCTION AND OVERVIEW

1.1 Introduction

This study seeks to investigate the techniques followed by three educators in their teaching of geometry in grade eleven. The participants in this study were the educators who teach mathematics in grade eleven and learners who took mathematics in the same grade.

This chapter is an outline of the study. The motivation for undertaking this research is discussed and the research questions are introduced.

1.2 Motivation for the study

In the years that I have been teaching mathematics, the part that gives learners most difficulty, is geometry. Often it is felt that the learners are weak and not the educators. On the other hand learners feel that this section of mathematics is difficult to such an extent that they would prefer that it not be taught in school. Informal discussions with learners as well as with some educators confirm the following: (a) learners have developed

a negative attitude towards euclidean geometry and (b) some educators find the euclidean geometry section difficult, especially those who have recently joined the teaching profession.

In responding to the above situation, the South African government called for a concerted effort to improve the teaching and learning of mathematics. Teachers are called to improve their mathematical content knowledge, while being provided with better learning support materials within a frame of revised curriculum structure and assessment procedures (Brown, 2002).

Being the head of the department of mathematics at my school I have witnessed situations where suitably qualified mathematics educators are scarce. In such cases the principals are forced to appoint unqualified educators. In the school where I teach we have three such educators. These educators did not study methods of teaching mathematics and their knowledge of mathematics leads one to doubt their ability to teach it adequately. The above situation may have adverse results to learners. This study is trying to question the effectiveness of the techniques educators employ in teaching euclidean geometry. It would also be interesting to know the level of satisfaction learners derive from these techniques.

Underachievement in mathematics (especially in South Africa) is of great concern and many authors have expressed it. The research conducted by the Human Science Research Council (HSRC) revealed that grade eight South African learners who took part in the Third International Mathematics and Science Study Repeat, ended up at the bottom of the list of thirty eight (38) countries (Howie, 1999).

National and international surveys of mathematics performance also revealed that secondary learners are unable to identify geometry shapes like rhombus, trapezium, kite and parallelogram (Triadafillids, 1995). The above situation also prompted me to find out the underlying cause of failure regarding the above fact.

The researcher, as the head of the department of mathematics at school, has observed that some educators still believe in the traditional way of teaching mathematics (especially geometry). The reason they give for teaching in the way they do, is that it saves them time and that they are able to cover a lot of work in a short time, thus are left with more time for revision.

The traditional way of teaching of geometry is based on the transmission of axioms and theorems formulated by other mathematicians. These are recorded in texts for learners to study. Learners are not given the opportunity to question and understand them. This creates an impression that geometry comprises the sequence of facts and formal proofs that should be followed as they are. Stodokly cited in Gourgey (1992) argues that the use of this method encourages students to expect to be told what to do and believe that they cannot discover on their own. He further states that the "explain- memorize" teaching method, which is prominent in traditional mathematics classrooms, promotes memorization and not understanding. Understanding is essential and crucial for success in mathematics, especially geometry. The concept understanding will further be discussed in the next chapter.

In an attempt to work towards resolving the problem above the researcher opted to undertake an investigation into the approach followed by educators in their geometry teaching techniques. The researcher is hoping to get to the roots of the reasons that cause this section of mathematics to be problematic. It is envisaged that once the reasons have been discovered the educators, researchers and other stakeholders would be able to work out other alternative techniques to the teaching of geometry in schools.

1.3 Personal Experience and Interest

In my teaching of geometry I have observed that if learners are well guided, they do enjoy learning geometry. My further observation has been that educators look at mathematics as a manipulation of numbers, using laws of algebra and less attention is put to spatial representation and language in the development and communication of mathematical ideas, which is essential in the teaching of geometry. Solving geometry problems in grade eleven relies on whether learners are able to visualize the problem situation rather than whether they know the proofs of theorems or not. However proofs of theorems are undeniably important. They should arise after the learner has made sense of the problem situation.

If learners are first exposed to activities that concentrate on individual figure recognition, production and naming, a sound basis upon which to progress to work at abstract level will have been provided. In high school one of the subjects I took was Technical Drawing. In this particular subject I was exposed to a variety of geometric figures, and my understanding of geometry was increased. This understanding resolved most of my problem that was related to the theorems in geometry. One example is the theorem, which states that perpendicular bisectors of the chord of a circle pass through the center of the circle. In Technical

Drawing this is basically saying that the point of concurrency of the perpendicular bisectors of the sides of a triangle (in this case the sides of a triangle are chords of a circle) is equidistant from the vertices of a triangle. This can be illustrated by constructing perpendicular bisectors of the sides of a triangle and the distance from the point where they meet and the vertex will be the radius of the circumscribed circle.

Drawing some of the geometric figures in Technical Drawing removes the suspicion of whether there is truth in the theorems and it encourages one to embark on the proof with all the necessary attention. Later in my teaching I discovered that the knowledge obtained outside geometry classes, like in a technical drawing class, adds more meaning to geometry. It also facilitates the understanding of essential properties of geometric figures. The researcher therefore feels that it would be useful to involve learners in more informal geometry that will enhance the necessary concepts in geometry before they are formalized as theorems and proofs. The décor of the mathematics classroom may also help in this regard. Mathematics classroom will be discussed in more detail in the next chapter.

In most cases learners are introduced to complicated problems too early, and usually they fail to solve them. This usually results in learners

thinking that all geometry problems are difficult and then lose interest in the subject. To some educators geometry problems are easy and they tend to give learners complicated problems at very early stage in the learning process. This practice discourages the slow learners and encourages the educator to attend to learners who are coping with work. I have also discovered that there is a lot of background information that learners are failing to connect to problems situations.

Learners in grade eleven have a lot of information necessary to solve geometry problems yet they struggle to solve them. The researcher feels that the challenge (for educators) lies in the choice of the best technique one should adopt to best assist learners. The aim should be to make sense out of the information they have already acquired in a more formulated discrete form. The lack of such a formulation further prompted the researcher to undertake this study.

This investigation attempts to interrogate methods implemented by (the three) educators in teaching geometry and the extent of their success. It is hoped that the information obtained from this study will contribute to the body of knowledge and the awareness to educators.

1.4 Current Teaching Techniques

Although the new system of education was introduced there has been little change in the way the learners are assessed in grade ten, eleven and twelve. The tests and examinations (even those set by the department of education) still reflect the past type questions. This situation is partly the reason why some educators still adhere to their old techniques of teaching since educators' style display examination orientated focus.

Discussions with educators who teach grade eleven revealed also that they had not shifted much from teacher-centered way of teaching. Most educators illustrate the proof of the theorem on the chalkboard. The educator will go through all the steps of the proof of the theorem while the learners sit attentively and looking at how it is done. An alibi to resort to a traditional way of teaching is the time constraint that governs the syllabus completion. Drilling in mathematics especially euclidean geometry is still largely considered as the best way of teaching (Slammert, 1991, pp 69). The investigative approach is seen as time consuming and delaying the process of completing the syllabus.

Outcomes-based education (OBE) among other things requires learners to: (1) Identify and solve problems, (2) make decisions using critical and (3)

creative thinking, and working effectively with others as members of a team (DoE, 2003). Educators channel learners to follow a particular way of solving problems. An educator does this unconsciously. The educator will after the proof of a theorem show learners how to use the theorem in solving a given problem. After this example the learners imitate the educator by solving similar problems.

I have observed in the school where I teach that even when the learners are given a chance to discuss a problem, they will recall the methods used by the educator in a similar situation. If they fail to remember they will quickly ask the educator for help. Rarely, will they try and sort the problem themselves. The educator interprets this behaviour as learners are participating in their work because they are talking to each other and with the educator. The problem arises when the educator is not with the learners or when learners are given homework. The educator will later realize that learners do not clearly understand the taught concepts.

If the above happens the educator usually will start all over again with what he/she did the previous day but this time spending most of the time illustrating to learners how the problem was supposed to have been solved. This procedure will go on until almost all the prescribed work in geometry

is done. The above situation will more often leave learners with the belief that, the method illustrated by the educator is the 'only' method, hence they will try to memorize the solutions with the hope that the same problems will be asked in class test or examinations, which does at times happen.

1.5 Focus of the Study and Research Questions

The focus of the study was on the techniques employed by the educators in their teaching of euclidean geometry in grade eleven. The study was therefore aiming at answering the following research questions:

- How did educators teach theorems and their proofs?
- What are techniques followed by educators in teaching learners how to solve geometry problems?
- How did learners perceive euclidean geometry in class?
- How did learners receive educators and their methods of teaching?

The following are sub-questions that the researcher felt would provide answers to the above research questions. These would be asked during the interview sessions with the three educators participating in the project.

1.5.1 Educators' Research Questions

1. How many techniques do you employ when you teach learners how to solve riders?
2. Which method/technique proves to be more successful and why?
3. How do you teach learners the following theorem?

The angle between a tangent to a circle and a cord drawn from the point of contact is equal to an angle in the alternate segment.

4. What in your opinion causes learners to fail in euclidean geometry?
5. On what learning theory/theories is your geometry teaching based?
6. Do you incorporate learning theory/theories in teaching?
7. If so how?
8. Do learners enjoy your geometry lessons? Please elaborate.
9. Are there any specific areas in grade eleven geometry which your learners enjoy success?

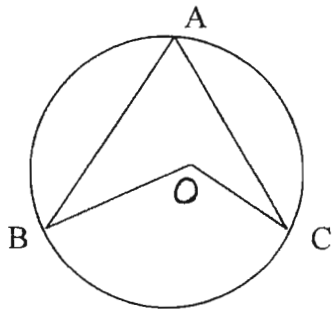
Learners from the participating schools (six from each school) were given the following questionnaire to answer as well as problems to solve.

1.5.2 Learners' Questions

1. How many techniques do you use in solving riders?
2. Which technique do you think is best for you? Why do you feel this way?
3. Solve the following problems and explain why you could /could not solve any of them:

3.1 Complete the statement of the theorem: "The angle between a tangent to a circle and chord----------."

3.2 In the following diagram O is the center of the circle and $B\hat{O}C = 124^0$: Work out the size of angle

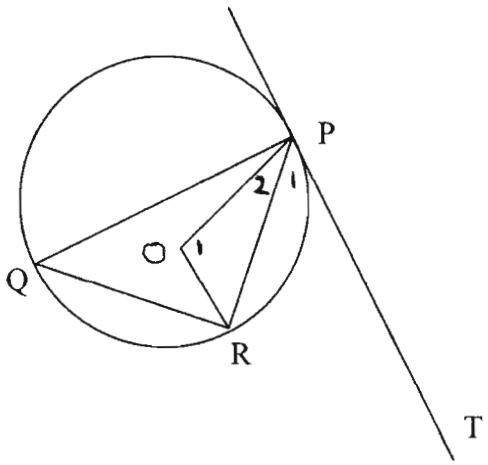


3.3 In the given diagram O is the center of the circle. TP is a tangent at P. Q and R are points on the circle. If $\hat{P}_1 = 40^0$, find the magnitude of angle:

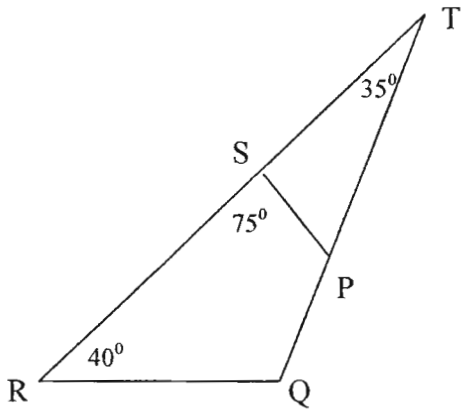
3.3.1 $\hat{Q}$

3.3.2 $\hat{O}_1$

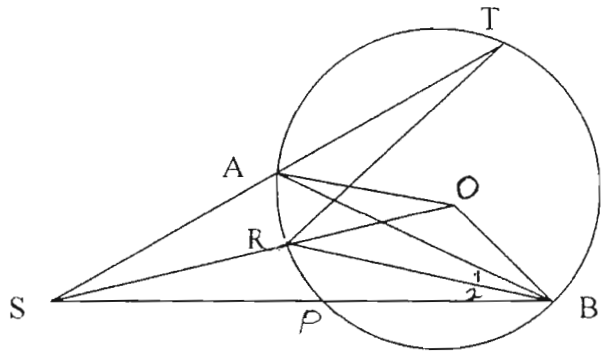
3.3.4 $\hat{P}_2$



3.4 Verify that PQRS is a cyclic quadrilateral in the following diagram.



3.5 In the following diagram O is the centre of the circle. TAS, ORS and BPS are straight lines. Prove that OASB is a cyclic quadrilateral if $\hat{B}_1 = \hat{B}_2$,.



4. What do you think are the causes of failure in euclidean geometry?
5. Do you enjoy geometry lesson? Why do you feel this way?
6. Do you enjoy the method your teacher employs in teaching you theorems?
7. What does he/she use to teach theorems?
8. Did you enjoy geometry in grade ten?
9. How would you compare your performance in grade ten to your performance now in so far as geometry is concerned?

1.6 The Significance of the Research

This study is important for the following reasons:

- It will contribute to the body of knowledge on the teaching of geometry in grade eleven.
- This study will add to literature concerning the topic. The researcher could not find enough South African material on the topic. The information gathered from the study will serve as the eye opener to educators and researchers.
- The research will contribute as a resource material to the upcoming generation of educators in their advancement towards seeking for new approaches in teaching, not only in geometry but also in other parts of mathematics.

1.7 Major problems and issues associated with research project

This study did not include all areas of learning euclidean geometry since it was not conducted in all grades (especially the grades lower than grade eleven). The researcher is therefore not in a position to know if the approach undertaken by the participants is used in other grades. The above is open for further investigation.

In order that the researcher got as much information as possible he had to:

- Build trust, between him and the participant.
- Be curious enough to dig as much information as he could possibly dig without arousing suspicion to the interviewee.
- Strive for naturalness; in order to secure what was within the mind of the interviewee.
- Foster a cordial relationship so that the presence of the interviewer would not hinder the accuracy of the responses.

Interviews and questionnaires were used as means to collect the data in this project. Questionnaires as means of collecting the data have limitations; ethically will always be an intrusion into the life of the respondent in terms of the time taken to complete the questionnaire. The level of treat or sensitivity of questions or the possible invasion of privacy, need to be tactfully addressed. Involvement in the research is likely to be a function of:

- Informed consent.
- The rights to withdraw at any stage or not to complete particular items in the questionnaire.
- Factors in the questionnaire itself (e.g. its coverage of issues, its ability to catch what respondents wants to say rather than to

promote the researcher's agenda) i.e. the avoidance of bias and the assurance of validity and reliability in the questionnaire issues of methodological rigor and fairness (Cohen et al, 2001).

The above limitation in this study was overcome by assuring participants of the confidentiality and freedom to withdraw at any stage of the project. The above was done in writing and signed by both the researcher and the participant.

Questionnaires limit the participant to the questions asked. This limitation deprives the researcher of the information that the participant might have and wish to share. Questions can be misinterpreted and thus change the meaning intended by the researcher. Similarly the researcher may interpret the response of a participant differently. Questionnaires nevertheless have advantages. They tend to be more reliable because they are anonymous, therefore encourages greater honesty (Cohen et al, 2001).

Learners were expected to respond on their attitudes towards the educator's approach in class. The educators on the other hand had to answer questions on their approaches and techniques. These were focused particularly in teaching learners proofs of theorems and solving geometry

problems. Responses from questionnaire could not guarantee that the respondent was giving a sincere response. Interview was therefore used to minimize the above limitation. The responses from the interview are more reliable because it involves the gathering of data through direct interaction between individuals.

CHAPTER 2

THEORETICAL FRAMEWORK

2.1 Introduction

This chapter will bring together views concerning the teaching and the learning of mathematics. The researcher is concerned mostly about the approach in teaching grade eleven geometry. It is therefore important to find out what others are thinking when it comes to the learning and teaching of mathematics (particularly, grade eleven geometry). This chapter will try to arrive at a general consensus of views on the teaching approaches in mathematics (especially in the teaching of geometry).

2.2 Constructing Meaning (Constructivism)

The constructive theory or model of learning suggests that knowledge is not often transferred directly from teaching to learner in the form that can immediately be understood. Researchers (e.g. Confrey, 1990; Hiebert & Carpenter, 1992) have shown from their studies that there are significant qualitative differences in the understandings that different learners develop in the teaching and learning contexts, and that it looks as if understanding is mostly different from what the educator intends.

Within constructivism there is a general consensus that learners' understanding usually has to be constructed by their own individual efforts as well as their own mathematical ways of knowing, as they strive to be effective by restoring coherence to the world of their personal experience (Cobb, 1994). I also concur with Orton and Frobisher (1996) that constructivism does not imply that learners can make progress only on their own, nor does it suggest that the educator has no contribution to make. The educator should create situations that will enable learning to take place and move away from spoon-feeding learners.

In order to realize the above, a social transformation should take place. In South Africa in particular the imperative to transform stem from a need to address the legacy of the apartheid in areas of human activity and especially in education (DoE, 2003). Education according to Taylor et al. (1991) is the need to democratize knowledge. Taylor et al. went on to point out that the essential of such democratization are that education must be accessible to all South Africans and that it must be relevant to the economics, social and political activities of its participants. Outcomes-based education (OBE) is the new South African system of education that has replaced the apartheid education system. Outcomes-based education

strives to enable all learners to reach their maximum learning potential by Learning Outcomes (L Os) to be achieved by the end of the education process. OBE encourages a learner-centred and activity-based approach to education. The L Os are built on the Critical and Developmental Outcomes that were inspired by the Constitution and developed through a democratic process (DoE, 2003). In mathematics the following L Os are to be achieved:

L O 1: Number and Number Relationships

L O 2: Patterns, Functions and Algebra

L O 3: Shape, Space and Measurement

L O 4: Data Handling and Probability (DoE, 2003).

In order to achieve the above L Os it is important for the educator not only to be concerned about the construction of meanings and understanding but also to understand the process of knowledge acquisition and knowledge transfer. The social interaction can play a measure role in the process of knowledge acquisition and knowledge transfer. The true direction of leaning and the development of thinking in our conception are not from the individual to the socialised, but from the social to the individual (Vygotsky, 1986, pp.36). Vygotsky (1978) also understood learning as the outcome of internal developmental processes that are able to operate only when the child is interacting with people in his/her environment and in co-

operation with his/her peers. The idea of zone of proximal development (ZPD) defined by Vygotsky (1978) as:

> the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers,

gives the explanatory framework for learning as a whole, both in the formal contexts, such as learning in schools, and in the informal contexts, such as learning outside schools in everyday situations (Newman & Holtzman,1993).

Vygotsky (1978) argued that the development of higher cognitive functions is launched within the ZPD, and that most learning within the ZPD takes place when learners get involved with tasks or problems which go beyond their immediate individual capabilities in which educators (or other adults) assists their performance, or in collaboration with more knowledgeable peers. Educators should therefore pre-define the kind of learning that will be achieved by the end of a learning process in terms of outcomes (Skinner, 1968).

Since this study was concerned with the approaches of educators in their teaching of grade eleven geometry, the focus was on the L O 3. To ensure

that learning will take place in order to achieve the above (L O 3), the educator should act as a facilitator of learning. Designing tasks, assignments, problems, projects and other activities, which will stimulate thinking and mental activity, can help in the achievement of L O 3. Moll et al (2001) concur with the above, they argue that learning is not spontaneous but is provoked by situations and by an educator.

One of the main approaches to mathematics teaching (that the new system of education is trying to eradicate) during the apartheid era was rote learning. Different people have different opinions why the above method was used. Adler (1991) among others pointed out that with only 12% of black secondary school educators having a degree, mathematics teaching by and large was tackled bravely by educators barely one step ahead of their learners. The above, Adler continues, resulted in authoritarianism and rote-learning methods predominance. Teaching of geometry, especially in grade eleven (in my experience) yields better results if learners are allowed to work in groups. Learners in groups share their own ideas in solving a problem. The level of contamination of people's thinking as a result of many years of domination has to be taken up seriously in the implementation of the National Curriculum Statement (NCS). People have tended to look down upon themselves especially the learners. Thus we as

educators need to encourage learners so that they appreciate that they can also create mathematics as their predecessors have done in the past. Volmink (1990) reiterate the idea that for so long, learners in mathematics classrooms has been socialized to believe that their own experiences, concerns, curiosity and purposes are not important. Mathematics is seen as being devoid of meaning, bearing no relevance either to their every day experience, or to the pertinent issues in their societies. Learning mathematics for these students partakes more of the nature of obedience than of understanding.

The social organization of the classroom is a fundamental part of learning geometry at school level. It should involve small groups who work together on the task at hand. Practicing group work develops communicating skills, pattern-seeking, generating and conjecturing skills. Learners are encouraged to develop ways of communicating their finding verbally in the task. This exercise involves learners in mathematical thinking. Since the work is done in groups and there is no single way of progressing through the task, learners can learn to co-operate, share ideas and discuss amongst themselves what they think and why (Adler, 1991).

In my observation learners working together in pairs or in small groups become effectively and constructively involved in a given task or investigation. A positive attitude to themselves and their ability to do mathematics develops. In geometry, learners feel comfortable to share their ideas with other learners and to take criticism in good spirit. It is through discussion and criticism among equals that effective learning will take place (Piaget, 1932). Discussion can assist learning at any level. The articulation of thoughts lays learners open for inspection, and criticism and the amendment that will thus lead to clarification and a coming together of understanding.

Studies have shown that children working together in dyads or triads tend to perform better or at a higher level than children working as individuals (Doise, 1990). The social constructivists' model is regarded as a socially constructed world that creates and is constrained by the shared experience of the underlying physical reality (Ernest, 1996). It is important therefore that the geometry educator structures his/her work in a way such that it allows learners to interact.

2.3 Understanding mathematics

Mathematics learning, especially in geometry should be based on understanding rather than on being able to repeat remembered routines and demonstrate particular basic skills. The above does not totally reject memory at the expense of understanding. It is true that some of the basic concepts need to be memorized in order to facilitate understanding. Orton and Frobisher (1996), ague that, in relation to memory, the more readily one remembers the easier it is to think. Success in geometry depends on how much one remembers basic concepts because it is from these concepts that proofs and solutions to geometry problems rely. Memory eliminates delay caused by searching for what can be likened to some missing piece of information. Less effort is required in pulling essential information to the forefront of the mind. Learners in geometry often find themselves in the above situation especially where they have to solve riders.

Learners, whom educators regard as being particularly intelligent, usually have swift and reliable retrieval systems, in that they recall things quickly and accurately. It is therefore clear that a good memory is an essential part in the learning and understanding of mathematics. Understanding helps one to construct meaning (in a given problem situation) using memorised

facts. It is therefore an essential requirement in the learning process. Orton and Frobisher (1996) divides understanding to two kinds, *instrumental* and *relational* understanding. The learning of many procedures of mathematics e.g. adding two fractions or multiplying two three–digit numbers is regarded as *instrumental* i.e. how to do, while on the other hand knowing why you do it is *relational.*

Some geometry problems require abstract thinking, understanding relational in this case becomes more relevant than memorized facts. Understanding relational will earn the learners more success in problem solving. It is therefore important that educators adopt teaching methods that will encourage learners to display their understanding of concepts. If learners learn with understanding, in the instrumented and relational sense, they should be able to move smoothly from concrete to abstract thinking at all stages of mathematics learning (especially geometry).

2.4 The role of an educator

An educator's responsibility is to design activities which will cause learners to participate actively in their learning (Frobisher, 1996). Further the National Curriculum Statement (DoE, 2003) of South Africa envisages an educator who among other things is a mediator of learning, interpreter

and designer of learning programmes and materials, and a leader. The Out Comes Based Education has seen a complete paradigm shift from previous traditional approaches used in South Africa, which was a 'teacher-centered' approach, to an educator who poses as a facilitator of learning.

Faulkner, Littleton and Woodhead (1998) described the traditional class as teacher-centered where the emphasis is on neatness, order and accurate reproduction of demonstrated procedures. In this class relationship during lessons was mostly restricted to that between the educator and each individual learner. Paper and pencil tests and percentage marks for achievement were the only form of assessment and report in mathematics. The implementation of the NCS changes all of the above. It requires learners to demonstrate an ability to think logically and analytically, as well as holistically and laterally. The learners are also expected to be able to transfer skills from familiar to unfamiliar situations (DoE, 2003).

Geometry in grade eleven develops from axioms, and theorems leant in lower grades. Educators in this grade are mainly concerned with the development of deductive reasoning and theory construction culminating in complete abstraction devoid of concrete interpretation (Dickson, Brown and Gibson 1984, pp. 19). It is in bridging the gap between concrete and

abstract that educators have an important role to play, and discussion between educator and pupils would seem to be essential.

Success cannot always be guaranteed, and therefore the educator must also expect to have to work hard in trying to move the learners forward in their mathematical thinking. In order for an educator to fulfill the demands of NCS successfully he /she should shift from traditional ways of teaching and adopt methods which will persuade learners to participate in their learning.

2.5 Mathematics classroom

Learners spend most of their time in the classroom. The classroom should therefore be made a pleasant environment in which learners would enjoy to spend their time in. Educators on the other hand wish that learners would be attentive to and involved in their schoolwork. Educators' expectation is known as psychological investment and has to be facilitated by educators (Wehlage et al., 1989). Educators do this by producing a positive atmosphere in their classrooms, through making lessons interesting and stimulating. They also provide a safe environment and appropriate support for learning.

The learning atmosphere alters learners in different ways. Each individual is influenced by classroom layout, seating, temperature and smell as well as the quality of learner-educator interaction in class (Chaplain, 2003). Mathematics learners are expected to think creatively. Classroom environment is crucial to the fostering of creative ability. An environment full of ideas, experiences, interesting materials and resources can stimulate creativity (Craft, Jeffrey & Leibling, 2001). The décor and organization of a classroom should transmit what one expects to be going on in class. The mathematics educator should link theories learnt in mathematics (especially in geometry) to the real world. In a mathematics class, posters, and three-dimensional objects on display are useful. They can arouse interest from learners and assist them in making sense out of mathematics.

The layout of the classroom also affects communication in class. Eye contact, social distance, posture and gesture can all be enhanced by attention to the classroom layout. Some learners can easily feel excluded because of where they are positioned in class. To avoid this happening, the educator should reflect on who is sitting where and the reason. This exercise develops a positive relationship with learners who are at risk of social exclusion. Where the individuals are asked to sit, the nature of work they are given, the degree to which they are empowered to ask questions

in class and emotional warmth of the class environment all have an influence on learners. The above influences how students think learn and feel about themselves and how they subsequently behave in class.

In conclusion, organizing the classroom directly influences both the nature of the interaction and the style of teaching and in addition should match the educator's behavioural goals.

2.6 Learning mathematics [teaching and learning]

Traditionally, mathematics teaching has relied heavily on exposition (by the educator together with the consolidation and practice by the learner) of fundamental skills and routines. Adler (1991) concurs with the above as she talks about her experience from dealing with students from 'white South African schools. This group of students as she describes it is a reflection of a presentation of mathematics as '... a body of knowledge that must be absorbed: questions, problems have only one answer and the object of the study is to get each answer right. This technicist approach to scientific knowledge produces students who are expects in memorizing and applying rules, but who struggle to step out of this narrow frame to make meaning of their knowledge".

Bazzini & Inchley, (2002) state that their research findings show that an educator's traditional view on mathematics and mathematics teaching is preserved and reflected in the teaching practice The study revealed that the adopted curriculum and limitation of time were two significant constraints. To deal with the above especially time constraint educators resorted to traditional ways of teaching. A group of preschool, primary school and high school educators according to Slammert (1991) concur with the above when they expressed their dissatisfaction concerning the teaching of mathematics. They said that they were convinced that the drilling method of teaching is still the best in that their learners will then know their work better and by heart. Traditional approach to teaching mathematics is generally easy to implement, educators are therefore tempted to adopt a traditional style of teaching.

Non-traditional approach, to teaching mathematics involves a number of innovations, which some educators will try to avoid. A non-traditional approach to teaching mathematics may involve more classroom activities, and thus encourage more student-to-student dialogue and other interactions. Therefore it will be more difficult for the teacher to maintain an order and discipline environment. Such discrepancies suggest that an educator's concern with the managerial aspect and his/her concern with

the completion of a specific content according to the schedule appeared to take precedence over his/her concern with the adoption of a relatively non-traditional mode of teaching. As a result educators choose a traditional approach to teach mathematics (Bazzini & Inchley, 2002).

The National Curriculum Statement (DoE, 2003) requires learners to develop the following:

- Critical awareness of how mathematical relationships are used in social, environmental, cultural and economical relations.
- The necessary confidence and competence to deal with any mathematical situation without being hindered by a fear of Mathematics.
- An appreciation for the beauty and elegance of Mathematics.
- A spirit of curiosity.
- Love for Mathematics.

To achieve the above, one would expect that a great deal of transformation is required. Educators might need to move away from predominantly old methods of teaching and adopt new methods of teaching. Discrepancy between the conception about mathematics contents and conceptions about mathematics pedagogy might be due to the ideas of mathematics teaching that are easier to verbalize. These ideas possess a considerable number of

non-traditional conceptions while their conception about the nature of mathematics remains traditional (Bazzini & Inchley, 2002). The philosophy of the present study agrees to a certain extent with the above study. Educators are aware of what they are expected to do but, for different reasons, they resort to mixing the traditional and non-traditional way of teaching. Some educators attribute failure to educators of previous grades and not to their methods of teaching. They claim that concepts which form the background to grade eleven works are poorly taught in lower grade. The educators are faced with a challenge of transforming themselves so that they will be relevant in the present education system.

The traditional method of teaching subscribes to what Osborne and Streatfield (2005) termed as *closed task.* An example of a closed task is when a learner is asked to solve a problem by following a prescribed sequence of steps. This action of an educator is based on the assumption that learners understood the theorems and therefore would use them to solve problems. The learners usually struggled. As the Head of Department of Mathematics at school I have observed many lessons like this and I am of the opinion that this practice should change.

Learners are expected to make decisions using critical and creative thinking. They are also expected to work effectively with others as members of a team, group organization and community (DoE, 2003). *Open task* as opposed to *closed task* described above, allow children to reach their full potential. In open task learners work in groups and are allowed to interact with each other. They are also free to choose any method of their choice to tackle the problem. If necessary the educator scaffolds their learning by using probing questions.

Open tasks promote talk and discussion between learners; it allows them to use mathematical vocabulary, justify and reason with each other. It will give learners more ownership and provide opportunities for them to show what they know rather than just what is asked of them. The open task helps learners to draw on prior learning and apply this to an unfamiliar context (Osborne and Streatfield, 2005).

Drawing from Orton and Frobisher (1996) geometry is the science of space, therefore children should become experimenters, exploring the properties and relationships of the space which is everywhere around them. Inquisitiveness of learners in grade eleven makes them natural investigators of physical objects. The educator's role would then be to

provide opportunities for this to occur, asking about and discussing with learners the properties of shapes and how they relate to one another. The constructivists view and the principle of the Outcomes Based Education emphasizes learners' involvement. This requires an educator to give learners more opportunities to explore and work on their own, while presenting him/herself as a facilitator. Educators should always be aware that they are now in charge of the learning of the child. Therefore they are expected to carefully design learning programmes and materials and to guide learners in order to ensure that they produce South African citizens who will be competent in a global market.

2.7 Learning and teaching geometry

The problem of geometry in grade eleven has resulted in some concerned educators trying alternative approaches to this section of mathematics. Coetzee (2003), reports that he had tried a different approach to the traditional way of teaching euclidean geometry. By the traditional way he refers to the approach that starts off with first explaining the theorem, followed by a few examples in application which culminate in the completion of an exercise based on the theorem and related axioms. The above contradicts the South African National Curriculum Statement (2003) which states that learners should communicate effectively using

visual, symbolic and language skills in various modes. This implies that learners should be exposed to various interrelated experiences which will encourage them to read, write and discuss mathematics (Roux, 2003).

Coetzee (2003) confesses that the persistent failure of learners to understand and to measure up to the demands of formal euclidean proofs has him reaching out for any alternative approach to teaching this section of the curriculum. This study is trying to find out the approaches followed by educators in teaching geometry in grade eleven. The findings from this study will in the light of Coetzee (2003) hopefully reveal problems faced by both learners and educators in the teaching and learning of euclidean geometry

2.8 Conclusion

The constructivists view the learner's understanding as being constructed by the individual's effort. The emphasis on the social interaction and communication is to facilitate learning. The educators should therefore create the environment which will allow learners to take advantage of social interaction.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

This chapter will introduce the methods adopted in the project. These will be discussed and the reasons for preference will be outlined. The issues connected to the research, the problems envisaged and proposed solutions will be discussed.

3.2 Research Method

The method employed in completing the project is the qualitative research method. The researcher collected the data by interviewing the participants. To supplement the information gathered from interviewing educators, learners were given questionnaires to answer. The researcher helped learners who did not understand certain questions.

3.3 Reasons for choosing the research methods

The failure rate in mathematics especially in geometry raises concern as it was explained in chapter one. The researcher therefore became interested in the techniques followed by

educators in teaching geometry in grade eleven. In order to find out more about the above, the researcher embarked on this study. The qualitative approach was chosen since it helps to gain more insight about the nature of a particular phenomenon (Leedy & Ormrond, 2001).

The project involved practices of different educators and the effectiveness of those practices. Qualitative methods were appropriate for this study because they were going to facilitate the study of issue in depth and detail (Patton, 2002). Qualitative research is descriptive therefore can be able to reveal the nature of the situation in the geometry class, including relationships between learners and an educator. It is also a great tool for discovering and interpreting existing problems (Leedy & Ormrond, 2001).

Educators are directly responsible for the learning in class. They are the ones who create learning situations for learners in class. Potentially educators will have lots of information that will be useful in this study. Interviewing educators would thus help to gather the required data. The advantage of interviews here was that they were open-ended; therefore permitted the researcher to follow

up leads and thus more information could be sought. This kind of flexibility is likely to yield information the researcher had not planned to ask for, and therefore enrich the findings in the project (Singleton & Straits, 1999).

In order to supplement the information gathered from interviewing the educators, questionnaires were given to learners. Questionnaires were used with learners because their responses would be anonymous. Learners may be more truthful than they would be in a personal interview especially when they discuss their educator (Leedy & Ormrond, 2001).

3.4 Sampling and Participants in the study

The researcher followed the Purposive Sampling method in selecting the participants in the study. Borrowing from Strydom et al. (2004) and Singleton & Straits (1999) this type of sampling is based entirely on the judgment of the researcher. The sample is composed of elements that contain the most characteristics, representative or typical attributes of the population. The schools chosen in the study are from the same ward and therefore the findings will be assumed to be most representative of the entire ward.

The participants in the study were drawn from three schools in the same ward. The following procedure was followed in selecting the participants.

- From each educator a list of learners according to their performance in the half-year examination was drawn. The list was then divided into three sections, the top, the middle and the bottom, and each learner in the list had a number assigned to his/her name.
- The researcher then cut pieces of papers with numbers assigned to learners. These were put into three separate containers. The first container had learners from the top section of the list; the second had learners from the middle section and the last one were learners from the bottom of the list.
- Two numbers were drawn from each container making up a total of six learners for each school.

3.5 Educators are always engaged in research work

Educators are always engaged in some kind of research as they prepare themselves for the next lesson. Educators do not submit their findings or experiences to any formal body. Nevertheless

their contact and informal discussions with personnel like subject advisors, senior education managers (SEM) and other educational officials, convey the message. The educators have first-hand information about the education of a child and how the child receives and responds to the body of knowledge presented to him/her by the educator. If taken seriously, teachers' research represents a radical challenge to assumptions about the relationships of theories and practices, schools and universities' partnerships and school structures, and educational reform (Smith & Lytle, 1993).

Educators' research is systematic and they derive intentional inquiry about their own work in class as well as in the school. Drawing heavily from Smith and Lytle (1993) "system" refers primarily to ordered ways of gathering and recording of information, documenting experiences inside and outside of classrooms, and making some kind of written record. Systematic is also defined as referring to ordered way of recollecting, rethinking, and analyzing classroom events for which there may be only partial or unwritten records. The term intentional indicates that educators' research is an activity that is planned rather than

spontaneous. Educators' research involves the collection, analysis and interpretation of data gathered from their own schools and classrooms, including journals, oral inquiries and studies.

Oral inquiries are usually written records of formalized inquiry procedures and other discussions convened for reflecting and questioning. The study was aimed at finding methods and approaches followed by educators in teaching geometry in grade eleven classes. In pursuing that, the researcher will hopefully learn how educators utilize their researching ability in teaching geometry. The researcher had a clear intention of finding out about the educators' techniques or methods. Therefore it was important to be systematic in the planning the research project, choosing the participants, as well as the methodology to be followed.

3.6 Focus of the study

The project was aimed at finding the techniques followed by educators in teaching geometry in grade eleven. This study was conducted in three schools in the Lower Umvoti Ward, Ilembe District in the Maphumulo circuit.

The sample comprised of the educators who are teaching mathematics in grade eleven as well as the learners who were doing mathematics in the same class. The researcher had hoped that some informed data would be found from the educators because educators are facilitators in teaching process. They are, on daily bases, consciously or unconsciously engaged in the research themselves and therefore are researchers in small ways.

3.7 The process followed in completing this study

- The researcher negotiated appointment dates with the participants
- The schedule of dates was drawn according to appointments
- On the agreed dates, the interviews were audio taped
- Learners were given questionnaires to answer and answers were collected on the same day of the visit.

3.8 Research instruments

For ethical reasons the participants and the researcher came to a common agreement about the confidentiality of the information to be collected in the project. It was agreed that no name of either

participants or school would be revealed and that the interviews would be audio taped. The pseudo names of both educators and the learners participating in the project were used in this study.

3.9 INTERVIEW QUESTIONS

3.9.1 Educators' questions

The following were the questions asked to the educators

1. *How many techniques do you employ when you teach learners how to solve riders?*

   This question was aimed at discovering if the educators were using different methods /techniques in their teaching.

2. *Which method /technique proves to be more successful and why?*

   This question will reveal to the researcher if the educator was able to handle learners with different abilities. It is an undeniable fact that educators are always faced with learners of different learning abilities. Therefore an educator should always be prepared to cater for these differences. When an educator chooses the teaching method he/she should take into account the diversities prevailing in class, which in this

particular study were cognitive abilities rather than racial abilities.

3. *How do you teach learners the following theorem?*

The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.

This question would reveal the specific techniques the educator followed when introducing the theorem to learners and the procedures the educator followed in teaching the above theorem and probable any other theorem.

4. *What in your opinion causes learners to fail in euclidean geometry?*

This question was aimed at revealing what educators believe to be the causes of failure in geometry, especially in grade eleven.

5. *On what learning theory is your geometry teaching based?*

The researcher believes that it is important to know and understand how a child matures cognitively. This information

helps the educator to choose the appropriate method and the teaching material he/she is going to use in his/her teaching.

6. *Do you incorporate learning theory/theories in your teaching?*

   It is not enough to know the theories but the educator should by all means try and infuse them in his teaching.

7. *If so how?*

   In cases where an educator had some knowledge of theories, this question was for the educator to explain how the theory was incorporated in his teaching for the benefit of a learner.

8. *Do learners enjoy your geometry lessons? Please elaborate.*

   This question was aimed at discovering the relations between the educator and the learners. Which relations would confirm the acceptability of the techniques employed by the educator in his teaching?

9. *Are there specific areas in geometry which learners enjoy success?*

   If the learners enjoy certain sections of geometry then the

researcher will inquire from the interviewee if those sections were taught differently from others.

3.10 LEARNERS

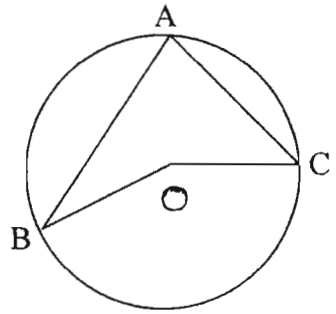
As it was explained earlier, the purpose of administering the questionnaires to learners was to confirm the data collected from the educators. The questions were therefore related to those used in the interviewing of the educators.

3.10.1 Learners' questions

The following questions were asked to the learners

1. How many techniques do you use in solving riders?
2. Which technique do you think is the best for you?
3. Solve the following problems and explain why you could /could not solve any of them:
    3.1 Complete the statement of the theorem: "The angle between a tangent to a circle and chord----------."

3.2 In the following diagram O is the center of the circle and $\widehat{BOC} = 124^0$: Work out the size of angle A

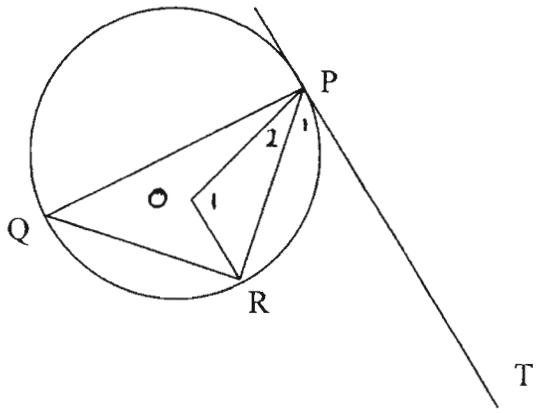


3.3 In the given diagram O is the center of the circle. TP is a tangent at P. Q and R are points on the circle. If $\widehat{P}_1 = 40^0$, find the magnitude of angle:

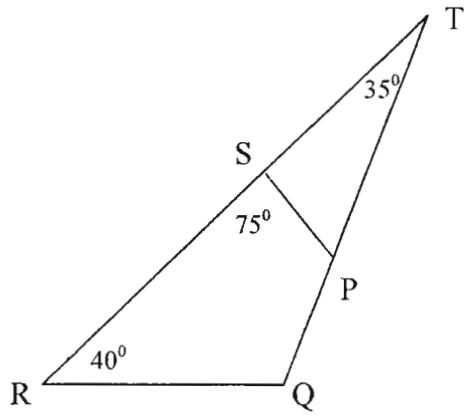
3.3.1 $\widehat{Q}$

3.3.2 $\widehat{O}_1$

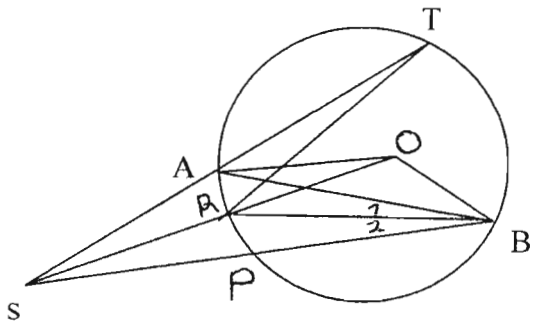
3.3.3 $\widehat{P}_2$



3.4 Verify that PQRS is a cyclic quadrilateral in the following diagram.



3.5 In the following diagram O is the centre of the circle. TAS, ORS and BPS are straight lines. Prove that OASB is a cyclic quadrilateral if $\hat{B}_1 = \hat{B}_2$.



4. What do you think are the causes of failure in euclidean geometry?
5. Do you enjoy geometry lesson? Why do you feel this way?

6. Do you enjoy the method your teacher employs in teaching you theorems?
7. What does he/she use to teach theorems?
8. Did you enjoy geometry in grade ten?
9. How would you compare your performance in grade ten to your performance now in so far as geometry is concerned?

3.11 Conclusion

The data collected from interviewing educators and learners responses from the questionnaires provided an understanding of how the three educators taught their learners geometry. The findings in the project will be discussed in details in the next chapter.

CHAPTER 4

RESULTS AND ANALYSIS

4.1 Introduction

This chapter deals with the findings in the project. The information was obtained using the methods described in the previous chapter. The researcher's intention was to learn more about the techniques used by the educators and how these techniques were applied in teaching grade eleven learners. Hence all efforts were made to obtain cosmetic free information.

4.2 Contextual Information

The study was conducted in three secondary schools in the Maphumulo Circuit in Lower Umvoti Ward. All the schools are in a rural setting. The learners were all black Africans and taught by black African educators. The majority of learners came from poor families, and most of them stayed with their grandparents. Most educators who teach in the area commute to school. The mathematics educators who participated in the study also commute to school. Learners travel long distances on foot to school everyday.

The mathematics classes in all three schools (where the study was conducted) ranged from twenty-five to fourty-five learners. In the schools visited, learners in general expressed love for mathematics. However, they all indicated that exercises in euclidean geometry were difficult to solve. On the other hand educators believed that learners find euclidean geometry difficult because they lacked the required mathematical background from the previous grades. One educator indicated that learners did not choose to do mathematics at will rather they were forced to attempt it.

In some cases (according to one of the interviewees) educators who label mathematics as a subject that contributes to high failure rate at school discourage learners. One of the educators who participated in the project commenting after the interview section said: "... *you find that some educators, they do say that this subject is decreasing the pass rate at the school, so why don't we get rid of it*". The same educator continued and said: *"at home learners do have a problem of being told that maths is difficult"*. The above may result in some learners go to mathematics classes believing that they will not make it because mathematics is a difficult subject. The performance of learners in geometry in this school will (according to the educator) be affected particularly because they did

not choose to do mathematics. The educator did not put it explicitly that he had contributed on the attitude of learners towards geometry. But the following response to the question that asked the educator to respond on whether learners enjoy geometry or not said it all. This response was: "*My problem in this school is that we don't choose learners to go for mathematics or other subjects, so not all the learners who are in mathematics class like mathematics ...*" My suspicion from the above response is that the educator who is teaching mathematics in this school did not take care of the learners who were straggling because they were not chosen by him. Comments from educators in this school, and some family members as well as the fact that learners do not choose to do mathematics, had (in my opinion) an adverse effect on both learning an teaching of mathematics.

The schools that participated in the study all followed a prescribed sequence of topics in mathematics. This was done in order to align the schools with the common tests set by the department and that added pressure to educators. As a result of the above, the educators rushed through the work and not enough time was given to each topic. "*... I was rushing time it was late towards the exams...*" was one of the responses from the interviewed educators. A learner from the same school as the

educator who gave the above response said: "*She use touch ups because she was late and she wanted to teach trigonometry*". The above responses give an indication that completing the prescribed work became more important then making sure that, learners understood what they were taught. The fact that the work programme that the schools were following in mathematics was rigid, some of learners were not fit for mathematics and time constraints, seem to be contributory factors to a low performance in mathematics especially euclidean geometry.

4.3 Teaching and learning euclidean geometry in class

Geometry classes are still conducted in a traditional way by some educators. Other educators however mix OBE methods with traditional methods. This study also showed that some educators do not use many methods of teaching learners how to solve riders. One of the educators when asked how many techniques she used in teaching to solve riders, the response was: "*I give them examples*". When this educator was asked to elaborate on that, she indicated that the examples were taken from the textbook and learners were guided through them. This particular educator seemed to lack creativity. She fits to educators that (Fashen 1990) described as graduates leaving on a special mixture of course and curricula that are scientifically and rationally planned and prepared for them by experts.

The study showed that almost all educators were using the same method, and they believed that in the process learners would learn how to solve problems in geometry. In some cases the difference would only be that, other educators picked problems from different sources like past examination papers, study aids and from different textbooks.

One of the interview questions required educators to explain how they taught learners a theorem that deals with an angle between a tangent and a chord. The following were responses of some of the educators regarding this question:

Educator 1. "One of the methods I use, to teach the theorem is to give them the drawing, and then you let them measure and let them to draw conclusions". This conforms to the constructivist general consensus that learners' understanding usually has to be constructed by their own individual effort (Cobb, 1994).

Educator 2: "What I did, I just used a telling method ...I took as it is from the book and then I have to write down the theorem and tell them how it goes, that step *kwenzeka njani* (how it is done)".

Educator 3: This educator did not teach the tangent chord theorem and justified it by saying that "*kwi*-Study & Master for standard grade -------- *leyo* theorem *ifika* as a corollary" (from the Study & Master for standard grade -----that theorem is presented as a corollary). This educator elaborated and said that he used illustrations on the chalkboard and gave different examples where the theorem could be used to solve problems.

The above two educators were practicing traditional methods of teaching in the sense that a teacher-centered mode of classroom organization was dominating. These educators were teaching by telling learners what to do and by illustrating on the chalkboard using different examples. The above is a good example of a traditional class where reproduction of demonstrated procedures and orderly predictable behaviour were highly valued (Faulkner, Littleton and Woodhead, 1998). One of the learners' commenting on how their educator was teaching them the theorems said the following: " *my teacher is perfect when it comes to teach theorems because he try by all means that the learners are listening to him"* The above comment confirms that some educators are still using traditional methods in their teaching.

Despite all the efforts by educators to teach, the learners in general are still not doing well in euclidean geometry. As both the educator and a researcher I became interested to find out more about the causes of this state of affairs. In an effort to find out more about the causes of failure in euclidean geometry both educators and learners were asked to respond on the following question. *What in your opinion causes learners to fail in euclidean geometry?*

The responses from learners revealed that they were divided on the issue of failure, some were blaming themselves and others were shifting all the blame to educators. The following were responses from learners to the above question:

Learner 1. *"I think it is because learners does not take care in geometry chapter and most of the learners they don't understand geometry"*

Learner 2. *"I think learners don't practice geometry"*

Learner 3. *"Misunderstanding and carelessness when answering questions"*

The above learners believed that they were not putting enough effort in their work and that as far as they were concerned were causes of failure.

The following set of learners had a different opinion; they felt that it was the educator who was responsible for their failure hence responded as follows:

Learner 4. *"I think because there is no teacher who has real knowledge to teach learners about geometry"*

Learner 5. *"Sometimes my teacher is teaching fast and theorems it very difficult if you leave no enough time"*

Learner 6 *"I think we don't understand that are causes of failure in euclidean geometry"*

Learner 4 gave an impression that he did not understand when the educator was teaching in class and he interpreted that as the lack of knowledge from the educator. It looks like both learners 4 and 6 had the same problem the only difference was that learner 6 acknowledged that they do not understand. Learner 5 failed to match the speed of the educator with the pace at which he was capable of absorbing what he was taught. This particular learner needed more time with an educator and it seem as if the educator was not aware that this particular learner was left behind. The important thing in terms of pedagogy is that people at different levels of mathematical understanding speak, use and understand terms differently (Wirszup, 1976). The educator should therefore be ready to cater for this diversity by preparing learning materials that will accommodate the

different levels of understanding in learners. The learners 4, 5 and 6 appear to be operating at a lower level of understanding than what the educator was thinking and the result was that these learners did not understand. It is therefore crucial that educators who teach geometry investigate their learners' understanding so that they will be able to provide meaningful learning experiences at the learners' particular level of development. Related to the performance of learners in grade eleven are the studies conducted in South African schools. They indicated that high school learners in general are still functioning more at concrete and visual levels than at an abstract level in geometry (Govender, 1995). The above is despite of the fact that the national school exit examination requires a clear understanding of underlying abstract processes. The above implies that at secondary schools learners in grade eleven are still functioning at a lower level of mathematical understanding, they speak, use and understand terms differently and yet educators use terms that can only be understood by learners who have progressed to the third or fourth Van Hiele level (Wirszup, 1976). It is therefore not surprising that learners will more often find themselves not understanding what the educator is trying to present to them. The latter is sometimes interpreted by some learners as lack of knowledge on the part of an educator, (see learner 2's answer to question 4 in the appendix).

In the question of problem solving, educators and learners agreed that most problems were encountered when learners had to use the theorems in solving the riders (geometry problems) especially if more than one theorem was involved in a problem. Educator1, responded as follows: "... *when they have to apply theorems in one drawing I think when this whole information has to be reproduced that's where they fail".* The response of educator 2 on the same question was: "... *they are using the reasons from this`* kule` *so they misuse the reasons and you'll find that all the procedure and rules* `basebenzisa` *of another theorem* `kwenye`". The response of the third educator was: "... *sometimes they simply forget which theorem to apply because they will be intergraded into one problem".* Learners gave responses which agreed with all the educators' responses (see responses number 6 of learners 1&2 of school C in the appendix). According to the three educators interviewed, learners solved problems successfully if they dealt with one theorem. The confusion began when more than one theorem was required to solve a problem. Learners, according to educators, tended to apply theorems in unfitting situations, that is, they used one theorem in the place of another. An obvious reason for the above would be that learners were unable to remember the theorem that will fit in a particular situation and that was more to do with memory. It is therefore necessary for educators to encourage learners to store some

basic concepts in their memories because the more readily one remembers the easier it is to think (Orton and Frobisher, 1996).

It appeared to me that the problem was with the designing of lessons; they did not allow flexibility to learners. Richards sited by Orton and Frobisher (1996), argued that learners will not become active by accident but by design. The implication of this argument is that for learners to actively involve themselves in their learning, educators should design lessons that demand learners to engage themselves fully. From the discussions with educators learners were given work so that they practice what the educator had taught them in class. The problems given were mostly taken from past examination papers with an aim of acquainting learners with the questioning style of the examination. Lessons were therefore not designed to allow space for reasoning and critical thinking, retrieval, understanding and use of information, relating learning to existing knowledge and experience. The poor design of lessons lead learners to spend most of their time trying to recall and reproduce the information from their educator instead of them using their creativity in working out solutions.

In this study there was no evidence of group work in class. Neither educator nor learners mentioned anything about group work in class situation. It was however found from learners that they did work in

groups during their study periods, especially when they were doing their home work. A confirmation by the educator in school A (about the above) was that learners group themselves for the purpose of completing their homework. The educator therefore had no direct control over those groups. Hence they were homogeneous in the sense that brighter learners were in one group and those who were not doing well, either worked as individuals or in their own groups. Group work promotes discussion and criticism among the equals which in turn results in effective learning taking place (Piaget, 1932). If the latter is not taken care of, learners will not fully benefit from the teaching.

It was my suspicion that the educators taught the way they did because they were either not aware or did not incorporate learning theories in their teaching. To address the above, the following question was asked.

On what learning theory/theories is your geometry teaching based?

All participants admitted that they were not using any theories and that they did not know any learning theory. The above was confirmed by the following responses:

Educator 3. "*...for me I'm just teaching without any theory because angikaze kahle, kahle ngiyithole iqualification yakwa teaching* (I never qualified as an educator)".

Educator 2. "to *be honest, No. What I do I focus on the subject as it is and on a belief that you got to move from simple to complicated, from concrete to abstract*".

The response of educator 2 revealed that this educator was following the theory of developing his teaching from familiar to unfamiliar situations or from known to unknown but was not conscious of it.

Educator1, of school A is a qualified educator, she responded by saying that she had no idea about the learning theories. When asked if she did take into consideration the mind of a child when preparing her lessons, she responded by saying that she took the learners as grown ups who are able to cope with the subject. This kind of response indicated to the researcher that this educator had never thought of how learners learn especially in euclidean geometry. The belief that learners in grade 11 are grown ups therefore can cope on their own, is contradictory to the constructivists' view that says that learners need educator's contribution in their learning (Orton and Frobisher, 1996). The above further implies that an educator should always be there to assist a learner in his /her learning.

Learners' acceptance or rejection of educator's method of teaching can be characterized by their attitude towards both the subject and their educator.

It is the researcher's belief that if the learners enjoy the lessons they will more likely do well in the subject. To address the above, the following question was posed to educators: *Do learners enjoy geometry lessons? Please elaborate.*

Responses from educators showed uncertainty. It appeared that they had never worried themselves about whether learners enjoyed their lessons or not. The responses like "*Ja- they do although I won't say they enjoy geometry...it is hard to say they enjoy geometry" (Educator* 1) and "*Yes they do, some of them, they do, but some of them you'll find that they have told themselves that No...geometry! But I have tried to give them attracting or interesting introduction of geometry and you'll find that they do"* (Educator 2) confirm uncertainty in educators. Educator 3 said that not all learners who did mathematics in class liked it, because they did not choose to do mathematics; they were picked and told to do mathematics. Consequently it was only those learners who were coping that enjoyed mathematics.

The general feeling of the educators in this study was that learners were more inclined to algebra rather than geometry. Responses of educators in all the schools that participated in the study confirmed the above. All

educators who participated in the study indicated that learners do better in problems that required them to calculate angles, than when they were asked to solve problems where they had to apply theorems. The next discussion on learners' perspective towards geometry and the learning of geometry serves as a confirmation of the above.

The first and the second questions were intended to find out about the techniques used by learners hence educators, in solving riders.

Question 1: How many techniques do you use in solving riders?

This question was asking learners to state the number of techniques they used to solve geometry problems in class.

Question 2: Which technique do you think is best for you?

This question required the learner to elaborate on the method that he/she preferred.

The tables below shows how the learners responded to the question that asked them to tell how many techniques they were using in solving geometry problems.

Table 1: Responses on the number of techniques learners used in class

		No of techniques per learner					
	School	**1**	**2**	**3**	**4**	**5**	**6**
	A	1	0	4	1	0	0
	B	2	2	1	1	0	0
	C	0	3	1	2	0	0
	Total	**3**	**5**	**6**	**4**	**0**	**0**

The following table shows the relationship between the number of learners (in percentage) and the number of methods they used in solving the problems.

Table 2: Percentage of learners using a specific number of techniques

	I	II	III	IV
No of techniques	1	2	3	4 & over
Response %	17	28	33	22

The results shown in table 2 above shows that 83% (column II, III and IV) of learners mentioned more than one technique. Out of the 83% of learners who said they used more than one technique, 55% (Column III and IV) of them said they were using more than three methods. It appears that learners who made up 55% did what came first in their minds, that is; they did not follow any specific method in working out the problems. At least 17% (Column I) of learners appeared to be using a specific method.

Seven out of eighteen learners who participated in the study (40%) were able to elaborate on their techniques. Out of seven learners who were able to elaborate on their techniques four of them preferred to calculate angles and three learners (who were from school A) responded as follows:

Learner 1: "It is to label the given figure before answering question. It become easily during answering questions."(See appendix)

Learner 2: "I prefer the method whereby I tick equal angles or supplementary angles etc. I prefer it because it makes it easier for me to attack the problem or questions that follow". (See appendix)

Learner 3: "My best technique is to look careful at the drawing first and tick the important points before I look at the riders". (*I think by riders the learner meant questions)*

The learners who gave the above responses were the only ones who were able to solve the problems given on the questionnaire. The other learners appeared to have no idea of what to do or where to begin if faced with problems in geometry.

The next section of the questionnaire dealt with problem solving. The learners (six from each school) were given three types of problems to solve.

- In the first type they were required to calculate angles.
- In the second type they were going to calculate angles and use what they find to prove if the quadrilateral in a given figure is a cyclic quadrilateral using mostly theorems and fewer calculations.
- In the third one they were going to prove the quadrilateral cyclic using mostly theorems and fewer calculations.

Tables 3 below, shows the number of questions that were correctly answered and the number of learners who answered those questions from different schools.

School A.

Two learners out of six were able to apply their knowledge of theorems and did correct calculations based on theorems to work out angles. One learner managed to calculate and got five out of six solutions correct. A fourth learner got two calculations correct whilst the last two did not get any calculation correct

Table 3: Summary of questions correctly calculated in school A

	Questions correctly calculated						**Total Correct**
Learners	**3.2**	**3.3.1**	**3.3.2**	**3.3.3**	**3.3.4**	**3.5**	
1	*	*	*	*	*	*	6
2	*	*	*	*	*	*	6
3	*	*	*	*	*		5
4	*	*					2
5							0
6							0
Total	4	4	3	3	3	2	19
Percentage	67	67	50	50	50	33	52

School B.

One learner got two calculations correct and three learners got each one calculation correct out of six. The last two learners were unable to get any calculation correct.

Table 4: Summary of questions correctly calculated for school B

	Questions correctly calculated						**Total Correct**
Learners	**3.2**	**3.3.1**	**3.3.2**	**3.3.3**	**3.4**	**3.5**	
1	*						1
2	*						1
3	*						1
4	*	*					2
5							0
6							0
Total	4	1	0	0	0	0	5
Percentage	67	17	0	0	0	0	14

School C.

Two learners out of six got each, one calculation correct. The other four learners were unable to get any answer correct.

Table 5: Summary of questions correctly calculated for school C

	Questions correctly calculated						**Total Correct**
Learners	**3.2**	**3.3.1**	**3.3.2**	**3.3.3**	**3.4**	**3.5**	
1	*						1
2	*						1
3							0
4							0
5							0
6							0
Total	2	0	0	0	0	0	2
Percentage	33	0	0	0	0	0	0.6

The first question in this section needed the knowledge of the theorem that dealt with the relationship between angles subtended by the same chord or arc, including the angle at the center. In the next question, (in addition to the relationship between the angles described above) the learners should know the theorem, which deals with the tangent and an angle on the alternate segment.

The results showed that 67% of learners in school A were able to solve problems involving the two theorems described above. This gives an impression that the educator's technique was successful; nevertheless 50% of learners were not doing well in cyclic quadrilaterals or had not yet mastered them. Results showed that 33% of grade eleven learners in this school could not answer any question on cyclic quadrilaterals.

In schools B and C learners were able to do problems that involved only the calculations of angles. No learner was able to work out correctly the problems on cyclic quadrilaterals. The discussion I had with the learners after they had completed the questionnaires revealed that educators did teach the learners theorems on the cyclic quadrilaterals. The conclusion that one can then draw from the above is that learners were not taught how to use the theorems in solving problems.

I also discovered that they were able to solve the problems when we worked together. That raised the suspicion that the techniques used by their educators in teaching them proofs and solving of problems related to cyclic quadrilaterals were not helpful. I felt that educators needed to spend more time in teaching learners how to apply theorems in solving problems rather than proving theorems. Unfortunately the educators usually do not

see this until after the learners have failed a test or examination.

Out of all eighteen learners who took part in the project three managed the calculations well, that is 17% of the participants. The three learners who made up the 17% are the same learners who were able to explain their techniques of solving riders. Six learners i.e. 33% of all participants managed to get at least one calculation correct and nine (50%) learners were not able to solve anything.

4.4 Geometry and learners

The development of an appreciation for the beauty and elegance of mathematics in class as one of the requirements of National Curriculum Statement (DoE, 2003), can be displayed by among other things:

- Learners' achievement in mathematics
- Life in mathematics classes, that is, the relationship between learners and between learners and educator
- Continuous performance improvement in the subject.

All of the above hinges on the educator's approach when teaching in class. The techniques used by the educator can promote or adversely affect the above.

To ascertain if the techniques used by educator did promote the achievement of the above, learners were asked to respond to the questions four and five from the questionnaire. The first question was concerned with failure in geometry. This question (question four in the questionnaire) was seeking to find out if learners were aware of anything that caused them to fail geometry in class. Some learners felt that the educators were responsible for their failure in euclidean geometry, while others felt that they were given too many theorems in one problem. The above has already been discussed earlier.

One of the learners' came up with an allegation that geometry was taught for the first time in grade 10 and that learners saw that as the reason for failure in this section of mathematics. The exact words were as follows: *"ingoba iqhala grade 10-ngabe kungcono ukuba iqala grade7".* (It is because it starts from grade 10, it would be better if it (euclidean geometry) begins in grade 7)

The above allegation was outside the premises of the project I therefore did not make any follow up on it; nevertheless, I feel this requires further research.

4. 5 Educators and learners in a geometry class

As it was mentioned earlier the relationship amongst everybody in a mathematics class can affect the performance especially in geometry. To address the above, learners were asked to give their comments on the geometry lessons, method used in teaching them as well as their general impression about euclidean geometry. In the question where learners were asked if they enjoyed geometry lessons, and the method used in teaching them, all of them responded by writing, "*Yes.*"

When one of the educators was asked about the teaching of a theorem on tangents, she said that she explained to learners how the theorem goes. After that learners were given problems to solve applying the knowledge of the theorems. Justifying her method of teaching the educator said she was rushing time. There was evidence that some learners were aware of the above. This particular problem was found in two schools (see learner 2 of school B and learner 2 of school C in the appendix). The responses from these learners gave a clear indication that learners were not given enough time to assimilate theorems.

It appeared that the learners (learners 2 from schools B and C) were not happy about the technique that their educators were using in teaching them

the theorems. Rushing through the geometry work was justified by one of the educators by saying that it was close to the examination and she wanted to go on to the next section of mathematics, the trigonometry. This kind of practice contributed to the poor performance, and a negative attitude towards geometry.

When it comes to methods that educators were using when teaching geometry, the study showed that mostly educators used the chalkboard and explained to learners what ever there was that the educator deemed should be explained. Here are some of the responses on this issue:

"My teacher is perfect when it comes to teach theorem because he try by all means that all the learners are listen to him", was a response from one of the learners in school C. From school B one of the learners gave the following response "*Ubhala ebhodini aphinde angichazele*". (He/she writes on the board and he/she also explains to me) and from school A, a learner said "He uses some of chalks and he tells us to not be stressed when we are studying mathematics.

The above further confirmed that indeed educators were still using traditional methods in teaching grade eleven geometry.

4. 6 Conclusion

All the schools that participated in the study were going to write the common tests. They therefore had the same timetable for these tests. The Department of Education in the province of KwaZulu-Natal had set these tests. The educators from the participating schools were following a prescribed sequence of topics in mathematics. The educators from these schools adjusted their pace of teaching to align themselves with the common tests times.

More often the educators whose schools are compelled to write these common tests (according to the educators who participated in the project) run out of time. They said that educators felt that the amount of work to be done before learners sit for test was too much. Educators had thus adopted techniques that helped them to complete the prescribed set work. The techniques they used in teaching euclidean geometry were successful in completing the work but unfortunately less helpful in teaching learners to solve problems.

When teaching theorems in euclidean geometry, educators drew diagrams on the board and reproduced the theorems from the book. Solving problems on the chalkboard with the educator taking a leading role was

followed by giving learners an exercise. Problems, which were used as examples, were taken from the textbook and some from the past examination papers. The responses from two learners who participated in the study showed that other educator taught their learners to mark the diagrams using their acquired knowledge in geometry. The study also revealed that learners who used the above method were able to solve problems in euclidean geometry. The results and their implication will be discussed in the following chapter.

CHAPTER 5

DISCUSSION

5.1 Introduction

This study was conducted in the Maphumulo area, in Ilembe District. It was the first time that research in this field was conducted. The results from this research emphasize the need for seeking better techniques of teaching euclidean geometry. The discussion of the results will concentrate on the main purpose of the study which was to investigate the techniques followed by educators in teaching euclidean geometry to grade eleven learners.

In order to obtain the required information it was necessary to subdivide the participants into two groups, namely, the educators and learners. The following discussion of the results will focus on the research questions underneath:

1. How did educators teach learners theorems and their proofs?
2. What are techniques followed by educators in teaching learners how to solve geometry problems?
3. How did learners:

- perceive euclidean geometry in class.
- receive educators and their methods of teaching ?

5.2 How educators teach learners theorems and proofs of theorems

Educators had difficulty in explaining the methods they used when they taught geometry in grade eleven classes. The educators that participated in the research were all asked to elaborate on their method of teaching. This discussion reveals that all educators that were interviewed were following the same approach of teaching theorems in class. The researcher expected that the educators would explain how they introduce learners to theorems as well as how they lead them to the actual proofs.

Responses from educators together with findings from learners clearly showed that there were difficulties in teaching theorems and their proofs. Educators reproduced theorems from textbooks and explained each step of the theorem. This was done while learners were listening attentively. This approach subscribes to the traditional way of teaching. Learners and educators do not benefit much from this approach. Maybe an approach similar to what Cobb (1994) describes as constructivism approach, where learners would be encourage to construct their knowledge and understanding using their personal experience, would be a better option. I

would expect that the educator would give a diagram to learners and by measuring and / or calculations learners would then draw conclusions. Most probably the conclusion that learners make would lead to a theorem.

Although educators followed mainly traditional ways of teaching there was some evidence that learners were sometimes given opportunity to construct their own knowledge. Out of the three educators who participated in the project, two of them (in our discussion) said that they sometimes did give learners a diagram and allow them to discover the theorem on their own. In my opinion the latter was what educators should have been doing more often. Not only because it subscribes to constructivism, but because it could train learners to solve even unfamiliar problems. The above became evident when learners participating in the study were given problems to solve. The learners from two participating schools where their educators were demonstrating to them how the theorems were proved were not able to use those theorems to solve problems. The learners who were given opportunity to work out theorems on their own and given guidance by an educator managed to solve problems from the questionnaire.

5.3 The techniques followed by educators in teaching learners how to solve geometry problems

Educators who took part in the study, because of the pressure from the common tests set by the department, find themselves drilling learners to solve similar problems. The above encourages learners to copy the educator's solutions to problems and practice them.

Piaget (1932) discussed in the theoretical framework in chapter two, emphasized that it is through discussion and criticism among equals that effective learning will take place. The educators however seem to prefer that learners be given more work that they will do on their own as individuals in class. The only time where the learners are given time to work in groups was (according to educators) when they prepare for examinations. These groups were formed by learners during the study periods and the educators were not in control.

Learning as seen by Moll et al (2001) is non-spontaneous but provoked by situations and by the educator. The above means that the educator is expected to design tasks, problems and activities that will stimulate thinking and mental activity to learners. Textbooks in mathematics have been there for some time and they are within reach of learners. Some

learners therefore tend to study the problems from the books and master them. When a different problem comes up in a test or examination they find themselves unable to solve it.

Educators in the study believed in what they termed as individual attention. After giving learners a problem or problems the educator moves around the class and assists those learners who indicate that they are struggling. Learners who do not shout for help are assumed to know what they are supposed to do. It will be after a test or examination that the educator will discover that no learning took place. Although the educators had no deliberate group work in their classes, learners revealed that they do help each other during the study periods and educators confirmed that. The participants both educators and learners agreed that discussions (among learners especially), results in more understanding. These discussions and exchange of ideas with their peers result in internal developmental processes taking place, which according to Vygotsky (1978) are produced by learning.

The learners' questions were subdivided into two sections. In one of the sections they were required to solve problems. The purpose of this section was to confirm the educators' techniques of teaching. Out of eighteen

learners who participated in the project only three learners were able to solve problems successfully. These were learners from the same school.

After the completion of questionnaires the researcher had a discussion with learners and discovered the following:

- Learners were failing to match the problems with a particular theorem.
- Learners were able to solve the problems if guided by questions.
- Learners became interested after realizing that they had enough information to solve problems.

The above is an indication that if the learners were guided properly and /or a different technique was used, they would be able to find solutions. The research showed that in most cases learners were not taught any specific method of solving euclidean geometry problems. Educators took problems from the textbook and past examination papers and taught learners how to solve those, and expected learners to figure out how to solve any other problem.

Three learners out of eighteen participants were able to describe the method they used when they solve problems. The same learners were able to solve the problems from their questionnaires. These learners were from

the same school, therefore they were taught by the same educator. The educator who taught these learners did not mention anything about the method described by these learners. Their method included marking important features of the diagram e.g. equal angles, equal sides parallel lines etc. before attempting to solve anything asked in the problem. This method was working for these learners, because they did manage to solve the problems.

One of the educators' interviewed said that he did use geometry instruments when teaching. It did not come out clearly however whether the purpose of using geometry instruments in his case was to teach, or to draw neat diagrams. In agreement with the above a learner from the same school as the educator above, said that their educator was using instruments. It did not come out clearly if the purpose of using instruments was to teach or not. Even if the educator was using the instruments to teach it was going to be a useless exercise since learners had no geometry instruments (according to the educator) and therefore could not practice. Experience in technical drawing taught me that some of the facts that are not apparent are easily illustrated through accurate drawing and measurements. It is therefore my feeling that educators should incorporate the use of geometry instruments in their teaching.

5.4 How do learners perceive euclidean geometry?

Although learners indicated that they were struggling with euclidean geometry, they did however express their interest in this section of mathematics. Responses such as the following:

- *" I think now I'm now excellent in geometry"*
- *" it is not too bad but it's fine*
- *"My performance is different to the performance in grade ten because I try to understand theorems first "*

give an indication that although learners are straggling in euclidean geometry but there is a step in a positive direction. What in my opinion needs to be done is to use the correct techniques in teaching. The techniques that the educator chooses should cater for all learners in his/her class and diversity in terms of level of understanding should be taken care of.

5.5 How do learners receive educators and their methods of teaching geometry?

Learning depends to some extent on how the learners perceive the educator who is teaching the subject. Chaplain (2003) argues that learning can be altered by the quality of learner educator interaction in class. Responses from learners in this study gave a general impression that those educators who were teaching grade eleven were managing to at least give

meaning to euclidean geometry. Learners seemed to lack the basic knowledge to build on in order to enable them to handle geometry in grade eleven. The grade eleven educators should design lessons which will develop in learners the basic skills necessary to solve geometry problems.

5.6 Relationship between learners and educators in class

The relationship between learners and the educator in class revealed that some educators are still following the traditional way of teaching. One of the questions asked to learner, required them to tell what the educator uses when teaching them theorems. The purpose of this question was to find out if the educators do use geometry instruments in their teaching. One learner responded by saying that the educator is perfect (by *perfect* in this context I assume the learner meant strict) when it comes to teach theorems. The learner continued and said that the educator tried by all means that all the learners are listening to him. Most learners in all three schools said that their educators wrote on the board and explained the theorem while learners were listening. All of the above agrees with a traditional class and the behaviour of a traditional educator as described by Faulkner, Littleton and Woodhead (1998).

The learners' comments about their performance in grade eleven, when compared to their performance in grade ten showed that some learners had confidence in their educators. Some learners indicated that they never did geometry in grade ten and they find it difficult in grade eleven. Other learners felt that they were beginning to understand the euclidean geometry better than when they were in grade ten. The majority of

learners in the study agreed that euclidean geometry was hard for them but nevertheless they were enjoying learning it and were slowly beginning to understand.

5.7 Summary

The system of testing seemed to have an input in the style of teaching that educators adopted. The demand by the department of education for improved results also added pressure to educators. The focus was thus on passing the examination in the expense of acquiring knowledge. Unfortunate the euclidean geometry demands knowledge, understanding and an ability to apply this knowledge in problem solving.

It was noted that educators who offer euclidean geometry in grade eleven were trying their best to teach learners. The educators gave an impression that they would not teach the way they were doing if the common tests were not there. The latter adversely affected their methods of teaching. It forced them to practice traditional ways of teaching. This was done (according to educators) because of the time constraints. They wanted to go through the prescribed work quickly so that when the time for tests came, the work would have been completed. Their teaching techniques were mostly teacher centered. It was noted however, that they did allow learner involvement in their teaching. They did create opportunities for learners to work on their own in class as well as during the supervised study periods.

CHAPTER 6

CONCLUSIONS

6.1 Introduction

The discussion in this chapter will evolve around the validity of the empirical data, the learner and educator in a classroom atmosphere, learners' inclusion as a source of knowledge and implications of the study for further research. Suggestions of answers to the questions in the first chapter are made. They will hopefully improve the way euclidean geometry is taught at school especially in grade eleven.

6.2 The validity of the empirical data

The participants in the study were the educators who are teaching mathematics in grade eleven and learners who were currently in grade eleven. It was therefore expected that the information that would be collected would be reliable. Participants especially educators tended to respond in their home language, isiZulu. Therefore some responses were translated into English. The relationship of learners and educators in a classroom situation had an impact in the data generated. The discussion below confirms the above.

6.2.1 Learner and educator in a classroom atmosphere

Educators in the study were in some cases defensive in their responses. They seemed to be more worried about learners in class and their performance. Educators would rather prefer to discuss their experiences in teaching learners than discussing their techniques of teaching. On the other hand, (although a few) learners tended to give responses that seek to reveal that educators were not teaching them well. The above conflict was an indication that in class, things were not as smooth as they should have been. The above behaviour may be attributed to the fact that some learners did not choose to do mathematics. The latter was indicated in chapter four. Nevertheless a reasonable number of learners saw improvement in their performance when compared to their previous experience. The above discussion revealed that it is important to choose the correct techniques to employ when teaching euclidean geometry.

The results drawn from this study showed that only a few learners are able to solve problems that need them to apply theorems. Almost all learners are comfortable with problems that require calculations.

6.3 Limitation of the study

The sample used in the study was small and that was the main limitation that impacts on the generalizing of the data. It is not certain that all educators teaching grade eleven especially in the Maphumulo area, are teaching like the ones that were interviewed. Another limitation was that schools, which participated in the study, were compelled to write standardized common tests set by the department of education. The latter affected the teaching techniques of educators in the study. The researcher is not sure whether the other educators from those schools who are not compelled to write the standardized test would teach the same way as educators interviewed. Hence generalizing on the teaching techniques is affected.

6.4 Involvement of educators and learner in relation to the nature of geometry

The interaction during the geometry lessons was mostly between the educator and learners. Educators took a leading part. They taught learners from the chalkboard; reproduce theorems and problems from textbooks and past examination papers. The only time where learners worked on their own was when they were given problems to workout as class work or homework.

Voluntary groups however, were formed which exposed learners to some aspects of constructivism. Opportunities for learners to interact and exchange ideas were thus created. The above approach works better for euclidean geometry, not only for the purposes of learning and teaching but also because geometric figures are all around us. We are constantly in touch with them and therefore everyone can contribute to the learning of euclidean geometry no matter how small a contribution is.

6.5 Implication of the study for further research

This study focused on:

- How educators introduce learners to theorems and the techniques they employ in class to teach learners how to prove a theorem
- The methods followed by educator in teaching learners to solve geometry problems
- Whether learners are able to use theorems and their proofs in solving problems in euclidean geometry.

The following are themes for further research:

- How can concrete objects be incorporated in the teaching of euclidean geometry in a classroom situation?
- How can educators be encouraged to use their creativity in

the designing of the activities that can promote more interaction among learners?

- How educators could make learners share their ideas and personal views during problem solving.
- To what extent can geometry instruments help in understanding principles and concepts of euclidean geometry?

6.6 Way forward

The poor performance displayed by learners in geometry is a worrying issue to all mathematics educators. This problem needs to be overcome. The following are some of the ways that can be put into practice to alleviate this problem.

- The awareness of the presence of euclidean geometry in the learners' environment should be developed.
- Encouragement of the use of skills imported from other learning areas e.g. from technical drawing.
- Learners should be encouraged to contribute in their learning by putting forward their views and by participating in group discussions. One of the Critical Outcomes of OBE requires learners to work effectively with others as

members of a team, group, organization and community. The classroom situation can provide opportunity to practice this requirement.

- Educators should allow learners to take a leading role in class by organizing activities that will provoke discussion amongst learners. It is from discussions among peers that learners' confidence is boosted and learning and mental development increased.

6.7 Summary

This study has attempted to find out the causes of failure in geometry. This was done by investigating the techniques followed by educators in teaching euclidean geometry in grade eleven. The discussion from this research is hoped to open opportunities for further research on the teaching of euclidean geometry especially at high school level. Educators, researchers and other stakeholders may want to scrutinize the suggestions made in this study for improvement in the euclidean geometry teaching techniques. Further research could be conducted to determine other ways in which geometry can be taught and produce better results.

REFERENCES

Adler, J. (1991). 'Towards democratic mathematics education in South Africa',in Julie C. (ed) People's Mathematics: Early Ideas and Debates. University of the Western Cape, Cape Town.

Bazzini, L. & Inchley, C.W. (2002). Mathematical literacy in the digital era; Research, teacher education and classroom practice aimed at mathematics education for all; *Proceedings of CIEAEM 53:* Verbania, Italy; July 21 – July 27, 2001.

Bester, G., & Budhal, R.S. (1995). The development of interest in mathematics amongst secondary school pupils; *Spectrum*, 33(1): (pp25 – 29).

Breen, C.J., Brown, M. & Gibson, O. et al, (1992). *Mathematics education for in-service and pre–service teachers;* Pietermaritzburg: Shuter & Shooter.

Brown, B. (2002). *Learning Maths in South Africa.* Achiever, 3: (pp.42 – 43).

Chaplain, R. (2003). *Teaching without disruption in secondary school;* A model for managing pupils' behaviour: London and New York: Routledge.

Cobb, P. (1994). 'Where is the mind? Constructivist and sociocultural perspectives on mathematical development'; *Educational Researcher, Vol* 23 *No* 7: (pp13 –20).

Coetzee, K. (2003). Teaching mathematical reasoning while teaching grade 11 euclidean geometry. *Proceedings of the Ninth National Congress of the Association for Mathematics Education of South Africa,* Vol 2: (pp 224 -225).

Coetzee, K. (2003). Teaching mathematical reasoning while teaching grade 11euclidean geometry; *Proceedings of the* 9^{th} *National Congress of the AMESA*. Cape Town. Woodstock. June 30–July 4, 2006.

Cohen, L., Manion, L. & Morrison, K. (2000). *Research methods in education.* London and New York: Routledge Falmer.

Cooper, H.M. (1979). 'Pygomalion grows up: a mode for teacher expectation, communication and performance influence', *Review of educational research,* 49: (pp 38 – 41).

Confrey, J. (1990). 'A review of the research on student conceptions in mathematics, science, and programming'; in Cazden, C.B. (ed) *Review of Research in Education.* Washington, DC: American Educational Research Association, 16: (pp 3-55).

Craft, A., Jeffrey, B. & Leibling, M. (2001). *Creativity in education;* London: Continuum.

Department of Education, (2002). *Revised national curriculum statement grade R – 9;* Pretoria.

Department of Education, (2003). *National curriculum statement grades 10 –12;* Pretoria.

Dickson, L. Brown, M. & Gibson, O. (1984). *Children learning mathematics: Spatial thinking;* Great Britain, Oxford: The Alden Press Ltd

Doise, W. (1990). 'The development of individual competencies through social interaction' in Foot, H. Morgan, M. and Shute, R. (eds) *Children helping children;* Chi Chester: John Wiley.

Ernest, P. (1996). 'Varieties of constructivism: A Framework for comparison', in L.P. Steffe, P. Nesher, P.Nesher, P. Cobb, G.A. Goldin & Greer, (eds)., *Theories of mathematical learning;* Mahwah, Lawrence Erlbaum Associates.

Fasheh, M. (1990). 'Community education: To reclaim and transform what has been made invisible 1993. From a dogmatic, ready-answer approach of teaching mathematics towards a community-building, process-oriented approach' in Julie et al. (eds) Political dimensions of mathematics education 2; Cape Town: Maskew Miller Longman.

Faulkner, D., Littleton, K., & Woodhead, M. (ed.) (1998). *Learning relationships in the classroom;* London and New York: Routledge.

Gourgey, A. F. (1992). Tutoring development mathematics: Overcoming anxiety and fostering independent learning. *Journal of*

developmental education, 15 (3): (pp 10 – 12)

Govender. M. (1995). *Pupil's proof-writing achievement in circle geometry.* Unpublished B.Ed. dissertation; University of Durban Westville.

Harris, M.J. & Rosenthal, R. (1986). 'Four factors in the mediation of teacher expectancy effects', in R.S. Feldman (ed.) *The social psychology of education: Current research and theory;* New York: Cambridge University Press.

Hiebert, J. & Carpenter, T.P. (1992). *'Learning and teaching with understanding', in Grouws;* D.A. (ed.) Handbook of Research on Mathematics Teaching and Learning. New York: MacMillan: (pp 65-98).

Howie, S.J. (1999). Third International Mathematics and Science Study Repeat (TIMMSR). Executive summery. Pretoria: Human Sciences Research Council.

Leedy P.D. & Ormond J.E. (2001). *Practical research planning and design*, 7th edition, Merrill Prentice Hall, Upper Saddle River, New Jersey Columbus,Ohio.

Magina, S. (1995). Investigating the factors, which influence the child's conception of angle; *Proceedings of the 19th Meeting of the International Group for the Psychology of Mathematic Education.* Brazil: Recife.

McCoy, L.P. (1992). *Correlates of mathematics anxiety: Focus on learning problems in mathematics;* 14(2): (pp 51-57).

Moll, I., Ggultig, J., Bradburg, J. & Winkler, G. (2001). *Learners and learning reader;* Cape Town: Oxford University Press Southern Africa.

Nakahara, T. (1995). *Children's construction process of the concepts of basic quadrilaterals in Japan;* Hiroshina University: Faculty of Education.

Newman, F. & Holtzman, L. (1993). *Lev Vygotsky: Revolutionary scientist.* London: Routledge.

Orton, A. & Frobisher, L. (1996). *Introduction to education: Insights into teaching mathematics*; Trowbridge, Wiltshire: Redwood Books Limited.

Osborne, A. & Streatfield, D. (2005). Mathematics Teaching. *Quartely Journal of the Association of Teachers of Mathematics.* 192: (pp35 – 36).

Patton, M.Q. (2002). *Qualitative research & evaluation methods;* London: Sage Publication.

Piaget, J. (1932). *The moral judgment of the child;* London: Routledge.

Roux, A. (2003). The impact of language proficiency on geometrical thinking. *Proceedings of the Ninth National Congress of the Association for Mathematics Education of South Africa.* Vol.1: (pp 362-363).

Slammert, L. (1993). Mathematical spontaneity: Innovative mathematics learning strategies for South Africa today. In Julie et al. (ed). *Political Dimensions of Mathematics Education 2.* Cape Town: Maskew Miller Longman.

Skinner, B.F. (1968). *The technology of teaching;* New York: Appleton Century Crafts.

Smith, M.C. & Lytle S.L. (1993). *Inside Outside: Teacher research and knowledge*; Teachers College, Columbia University.

Strydom, H, Fouché C.B & Delport C.S.L. (2004). *Research at grass roots. For the social sciences and human service professions;* Hatfield Pretoria: Van Schaik Publishers.

Singleton, R.A. & Jr., Straits B.C. (1999). *Approaches to social research;* New York: Oxford University Press.

Taylor, N., Adler, J., Mazibuko, F. & Magadla, L. (1991). People's education and the role of mathematics: *A workshop. Julie, C. (ed) People's mathematics: Early ideas and debates.* University of the Western Cape. Cape Town.

Triadafillidis, T.A. (1995). Circumventing visual limitations in teaching the geometry of shapes. *Educational studies in mathematics*. Vol 29: (pp 225 – 235).

Vygotsky, L.S. (1978). *Mind in society: the development of higher psychological processes;* Cambridge, MA: Harvard University Press.

Wehlage, G.G., Rutter, R.A., Gregory, A., Smith, N.L. & Fernandez R.R. (1989). *Reducing the risk: School as communities of support;* Lewis: Falmer Press.

Williams W.V. (1988). Answers to questions about mathematics anxiety. *School science and mathematics. Vol 88 No 2*: (pp 95 – 104).

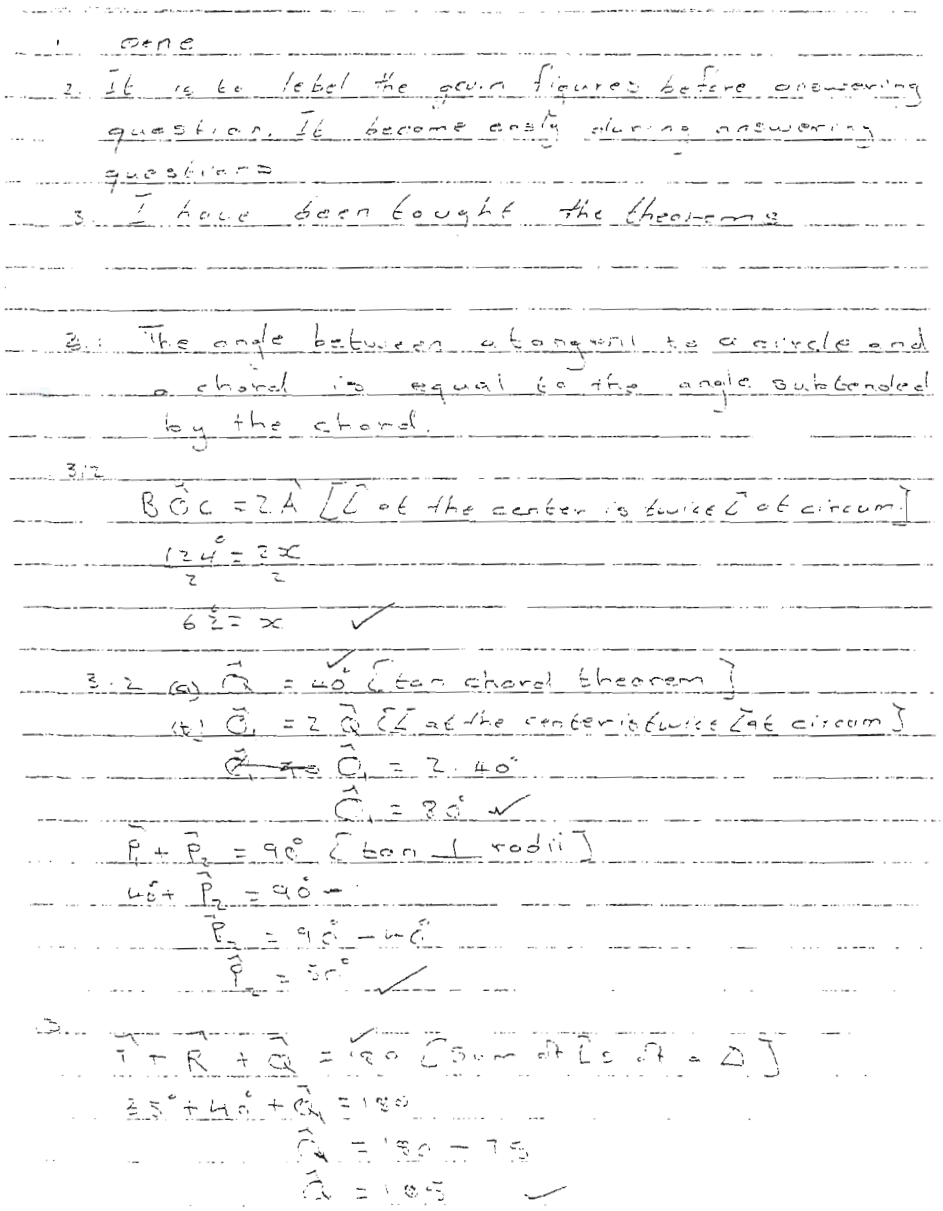
Wirszup, I. (1976). Breakthroughs in the psychology of learning and teaching geometry. In J.L. Matin (Ed.), *Space and Geometry: Paper from a research workshop (*pp. 75-97). Colombus, Ohio: ERIC.

APPENDIX

Learners' written responses and worked answers

School A

Learners 1



1. none
2. It is to label the given figures before answering question. It become easy during answering questions
3. I have been tought the theorems

3.1 The angle between a tangent to a circle and a chord is equal to the angle subtended by the chord.

3.2 $B\hat{O}C = 2\hat{A}$ [∠ at the center is twice ∠ at circum]

$\frac{124^\circ}{2} = \frac{2x}{2}$

$62^\circ = x$ ✓

3.2 (a) $\hat{C}_2 = 40^\circ$ ✓ [tan chord theorem]

(b) $\hat{O}_1 = 2\hat{C}_2$ [∠ at the center is twice ∠ at circum]

$\hat{O}_1 = 2 \cdot 40^\circ$

$\hat{O}_1 = 80^\circ$ ✓

$\hat{P}_1 + \hat{P}_2 = 90^\circ$ [tan ⊥ radii]

$40^\circ + \hat{P}_2 = 90^\circ$

$\hat{P}_2 = 90^\circ - 40^\circ$

$\hat{P}_2 = 50^\circ$ ✓

3. $\hat{T} + \hat{R} + \hat{Q} = 180^\circ$ ✓ [Sum of ∠s in Δ]

$35^\circ + 40^\circ + \hat{Q} = 180^\circ$

$\hat{Q} = 180^\circ - 75^\circ$

$\hat{Q} = 105^\circ$ ✓

Learner 2

1. 3

2. I prefer the method whereby I look equal angles or supplementary angles etc. I prefer it because it makes it easier for me to attack the following questions that follow.

3.1 The angle between a tangent to a circle and a chord equals to the angle subtended at the circumference by a chord. ✓

3.2 $B\hat{O}C = 2x$ ..... [$\angle$ at $\odot$ is $2\angle$ at circ...]

$\frac{124}{2} = \frac{2x}{2}$

$62^\circ = x$ ✓

$x = 62^\circ$

I solve these problems because I will understand them.

3.2 a) $\hat{Q} = \hat{P}$ ..... [tan-chord theorem]

$\hat{Q} = 40^\circ$ ✓

b) $\hat{O}_1 = 2\hat{Q}$ ..... [$\angle$ at $\odot$ is $2\angle$ at circ...]

$\hat{O}_1 = 2.40^\circ$

$\hat{O}_1 = 80^\circ$ ✓

c) $\hat{R} = \hat{P}_2$ ..... ✓ [Base $\angle$s of isos $\triangle$ OR = OR]

$\hat{P}_2 + \hat{R} + \hat{O}_1 = 180^\circ$ ..... [$\angle$s in $\triangle$ QPR]

$\hat{P}_2 + \hat{P}_2 + 80^\circ = 180^\circ$

$2\hat{P}_2 = 180^\circ - 80^\circ$

$\frac{2\hat{P}_2}{2} = \frac{100^\circ}{2}$

$\hat{P}_2 = 50^\circ$ ✓

I also understand ~~them~~ these problems

3.3 $\hat{T} + \hat{R} + \hat{Q} = 180^\circ$ ..... [$\angle$s in $\triangle$TRQ]

$35^\circ + 40^\circ + \hat{Q} = 180^\circ$

$\hat{Q} = 180^\circ - 75^\circ$

School B

Learner 1

1. We use only 6 riders.
2) Teni Technique two because to me it is very easy to understand.
3) The angle between a Tangent to a circle and chord is called radial
3.2 The size of x will be 56°
3 $\hat{Q} = 40$ ✓
$Q^1 = 40$ ✗
$P\hat{2} = 100$ ✗
3.3 $P\hat{Q}RS = 150° + 30 = 180°$ $P\hat{Q}R\,S = 150°$
$P\hat{T}Q = 30°$
$P\hat{Q}R = 180°$
3.4 ~~if $B\hat{1} = \hat{B}2$~~ if $B\hat{1} = B\hat{2}$
~~$OAS2 = 180°$~~ $OAB = B\hat{1}B\hat{2}$
$= OAB = 180°$

NB: 4. I think we as learners have no idea about Geometry.
5. Yes I enjoy Geometry because is very easy and understandable
6. Yes he is very Good in teaching Mathematics Especially geometry.
7. He used to teach us about Pythagoris or Radials.
8. No: because I learned Geometry in a very Short Period of time
9. Ee Sir I'm now started to be little bit clear about Geometry.
10. Because the teacher who teaches us Maths has got a potential to teach us in a polite manner so that we can heared what are we going to do.

Learner 2

1. I think they are three techniques
2. Is the first technique because you solve 90° only and using numbers below 90°
3.
3.1 The angle between a tangent to a circle and a chord is equal to a circumference of an angle chord.
3.2 The size of x of $B\hat{O}C = 124°$ is 62° ✓ and BC is arch
a) $\hat{C} = 60°$ ✓
b) $\hat{D}_1 = 150°$ ✗
c) $\hat{R}_2 = 45°$ ✗
4. RQ is tangent, SP is chord, $\hat{Q} = 90°$
5. $\hat{A}_1 = \hat{S}$ and $\hat{O}B = OA$ and $\hat{S}_1 = \hat{S}_2$ and Interior A = Interior P
6. The educators who not give them their time to educate childr
7. No because Geometry is challenging too much.
8. No. because she did not give hers eti time to make us cl about theorems.
9. She use touch ups, because she later and she wanted to tea
10. Trigonometry
11. No.
12. My performance in grade 10 is so bad and now I try my best to know exactly what is Geometry.

School C

Learner 1

3.4 $\hat{B}_1 = \hat{B}_2$

Prove: $\hat{A} + \hat{P} + \hat{R}$ (are $\triangle$ base Angles)

$\hat{B} = \hat{A} + \hat{P} + \hat{R}$

$\hat{B}_1\hat{Q} = \hat{B}_2 + \hat{B}_3$

$\therefore \hat{B}_1\hat{Q} = \hat{B}_2 + \hat{B}_1$

$\hat{B}_1\hat{Q} = \hat{B}_2$

E

5) I don't enjoy maths Geometry

a) I don't enjoy it because they are be many theorem that we are teaching it

7) Some time she give us work and I use it just like that.

8) In grade 10 I was enjoying the geometry but now in grade 11 I'm starting to get bac Even in the test I lost fail

9) The performance of grade 10 it was the best. Even the teacher he was the best teacher but now the knowledge that I have is so few for nothing in the Geometry.

Learner 2

1. THERE are three techniques.
2. THE techiques that I think is best to me is solving x.
I feel this because it is easy to see it
3.1 Is Equal to 180°.
3.2 $B\hat{A}C = 2B\hat{O}C$
$x = 2 \cdot 124°$
$x = 248°$

3.2 $\hat{Q} = 200°$
$b_1\hat{O} = 100°$
$\hat{P}_2 = 40°$

3.3 it because it have four sides
3.4. $\hat{B} = \hat{A}$

4. I think because there is no teacher who ~~have~~ have real knowledge to teach learners about Geometry.
5. ~~Yes~~, I feel Right but if one drawing mix all theorems & having a problem that what can I do.

6. NO because sometimetimes my teacher is teaching fast and theorems it very difficult if you have no enough time.

7. He/she use to teach by using her information without using a Mathematics book

8. ~~Yes~~ I did enjoy it