An Exploration of Pre-service Science and Mathematics Teachers' Use of Visualisation in a Problem Solving Context: A Case Study at a South African University

By

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Abstract

The poor performance of learners in Science and Mathematics in South Africa is a persistent cause for concern to stakeholders in education, and to society at large. Teacher training institutes form crucial stakeholders in Science and Mathematics education. This has been the underlying motivation for this case study, which is based on an exploration of pre-service Science and Mathematics teachers' use of visualisation within a problem solving context. The study is grounded in the interpretivist paradigm. The purpose of this study stems from anecdotal evidence that has showed teachers' reluctance to teach problem solving because they are unequipped and/or not confident in solving problems.

The exploration of pre-service Science and Mathematics teachers' use of visualisation in a problem solving context revolved around the following critical questions: 1. What do pre-service Science and Mathematics teachers understand by problem solving within a visualisation context? 2. Why do pre-service Science and Mathematics teachers choose to use the visualisation strategies they use when teaching problem solving? 3. How do pre-service Science and Mathematics teachers plan the use of visualisation when preparing their lessons? The framework used to guide this study falls within the interpretivist paradigm and the theory used is the metacognition theory. This theory refers to a higher order of thinking and, simply put, thinking about thinking. In this study, it was analysed how pre-service teachers view their teaching and what their understanding of visualisation is within a problem solving context.

The pilot group comprised five pre-service Science and Mathematics teachers at a South African teacher training institute who were registered for two modules, namely Natural Science Method Two, and Mathematics Method Two. These modules include the teaching of problem solving. A purposive sample population of eighty pre-service teachers were invited to participate in this project, and twelve completed part of the project, while five pre-service teachers participated until the conclusion of the project.

A qualitative methodological approach was used and pre-service teachers participated in four stages of data collection. Firstly, a semi-structured questionnaire was used to collect the biographical data of the participating pre-service teachers, and their understanding of problem solving and visualisation. Secondly, a task sheet was administered, which included a Science as well as a Mathematics selection of problems for the pre-service teachers to solve. All problems were purposively selected because visualisation methods could have been used to solve them. This tool was used to decipher what visualisation strategies pre-service teachers use when solving problems and why they use these strategies. Thirdly, a lesson plan was developed by participants to enable an exploration of how they taught problem solving using visualisation, as well as what cognitive processes they used to incorporate visualisation into problem solving. The fourth stage involved engaging participants in individual, face-to-face interviews. Semi structured interview schedules were used for both interviews. All responses were analysed and focused on the three research questions. The findings revealed that the majority of the pre-service teachers understood visualisation as a set of teaching aids that made solving problems easier. The majority of participating pre-service teachers solved Mathematics problems accurately when they used a combination of diagrams and formulae. The responses to the Science problems revealed that the majority of participating pre-service teachers used formulae instead of diagrams to solve them. However, the opposite scenario was presented by these participants when they generated their lesson plans. A greater variety of visualisation strategies were used in the Science lesson plans than in the Mathematics lesson plans.

The findings show that the use of visualisation in problem solving helped pre-service teachers solve Science and Mathematics problems successfully. It is anticipated that the pre-service teachers will take this finding and make use of it in their classes in the near future, which should in turn develop more competent problem solvers at schooling level.

DECLARATION

I, Levashnee Govender declare that:

(i) The research reported in this thesis, except where otherwise indicated is my original work;

(ii) This thesis has not been submitted for any degree or examination at any other university;

(iii) This thesis does not contain other persons' data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other persons;

(iv) This thesis does not contain other persons' writing, unless specifically acknowledged as being sourced from other researchers. Where other written sources have been quoted, then:

a) Their words have been re-written but the general information attributed to them has been acknowledged;

b) Where their exact words have been used, their writing has been placed within quotation marks, and referenced.

(v) The work described in this thesis was carried out in the School of Education, University of KwaZulu-Natal from 2012 to 2013 under the supervision of Dr. R. Mudaly and Dr V Mudaly; and

(vi) The Ethical clearance No. HSS/0194/015M was granted prior to undertaking the fieldwork.

Signed: __________________________ Date: ______________________

As the candidate's Supervisors I, Dr Ronicka Mudaly, and I Dr Vimolan Mudaly agree to the submission of this thesis.

Signed: __________________________ Date: ______________________

Signed: __________________________ Date: ______________________

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I dedicate this labor of love to:

My Alpha parents Mr Vigeon and Mrs Rajis Govender, to make you both proud is all I live for;

Lord Ganesha, for everything that I am and carrying me through this task. I hope I am a worthy devotee;

My brother Pavendren, with your caring and supportive nature I sometimes forget you are younger than me;

My sister Maveshnee, you are a great example to follow. Thank you for proofreading and assisting me when I needed it;

My darling husband Revashen, thank you for taking over as chef for as long as I needed and thank you for your daily words of encouragement and reminders that this is just a stepping stone to greater things I can achieve.

Lastly, to my babies Rex, Mia, Kloë; and Zara, you didn’t need words to show your love and understanding.

To a family who takes on the world every day and wins.

DEDICATION

To my Parents with Love

To my Supervisors with Love and Appreciation

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CHAPTER 1 INTRODUCTION

1.1 Introduction

This chapter presents a specific problem under study and the research methods used to study it. A brief but concise overview of the study is also provided.

1.2 Background to the problem

This study was conceived from my personal experiences with Science and Mathematics learners with respect to problem solving. Anecdotal evidence convinced me that learners struggle with problem solving because many teachers do not model efficient problem solving strategies. It is possible that the absence of visual strategies may be the main issue that inhibits learners from effectively solving problems.

The Revised National Curriculum Statement (RNCS), and Curriculum and Assessment Policy Statement (CAPS) in both Natural Science s (Department of Basic Education, 2011, p.7) and Mathematics (DoBE, 2011, p.12) emphasise the importance of learners developing their skills in both problem solving and visualisation. Despite this requirement in the RNCS and CAPS documents, it is my opinion, based on my experiences as a practicing Mathematics and Science teacher that many Science and Mathematics teachers fail to adequately address issues related to problem solving and visualisation. In dealing with problem solving, multiple representations should be explored because different methods may appeal to the varied intelligences of different learners. Multiple representations are different methods of representing the same problem and its solution. Simply put, they are used to explain and solve a problem in different ways. Multiple representations include graphs and diagrams, tables and grids, formulae, symbols, words, gestures, software codes, videos, concrete models, physical and virtual manipulatives, pictures, and sounds. The preceding examples of the different methods of representation reveal the visual aspects which can serve as cognitive tools.

It may be argued that of all the senses, sight is the most important when dealing with problem solving. Therefore, it can be posited that the skill of visualising is one that teachers need to instil in learners. In saying this, one would presume that teachers themselves understand what visualisation is and the strategies it involves. This forms the basis for the rationale of this study, which focuses on how pre-service Science and Mathematics teachers teach problem solving using visualisation.

It is assumed in this study that by using different ways of representing a problem, especially visually, a teacher is more likely to help more learners understand and solve Science and Mathematics problems. This study argues that using multiple representations for a problem is similar to using different lenses through which learners can look at a problem. One lens may suit particular learners better than others. In this study the use of multiple representations supports and requires tasks that involve decision making and other problem solving skills. This study contends that the use of multiple representations has many advantages in aiding learners towards a deeper understanding of problem solving, and its effectiveness depends on how well teachers present the concept. Pre-service teachers need to understand and appreciate the value of multiple representations in order to practice it. The view held in this study is that being a teacher means being creative in helping learners to understand concepts that are presented to them; often this means being able to explain a single problem in different ways.

The idea of using different ways of presenting a problem is linked to the theory of multiple intelligences. Problem solving questions do not merely test a single concept, and in turn a single ability, but rather multiple concepts and multiple abilities (intelligences). The concept of multiple intelligences was theorised by Gardener in 1983 and is summarised by Armstrong (2011) as: “Linguistic intelligence ("word smart"); Logical-mathematical intelligence ("number/reasoning smart"); Spatial intelligence ("picture smart"); Bodily-Kinesthetic intelligence ("body smart"); Musical intelligence ("music smart"); Interpersonal intelligence ("people smart"); Intrapersonal intelligence ("self-smart"); Naturalist intelligence ("nature smart")”. It is the task of the teacher to help develop as many intelligences in their learners as possible. The use of multiple representations may help to do this because it opens many opportunities for a teacher to ‘reach’ his or her learners. It can therefore be said that teachers can enable their learners to solve problems by igniting the different intelligences that their learners possess, (Armstrong, 2008).

In my view, a useful method for looking at teachers’ visual strategies would be to examine pre-service teachers’ knowledge of visualisation strategies. This requires that attention be paid to the pre-service teachers’ education curriculum and in particular, the focus of problem solving using visualisation. One could presume that pre-service Science and Mathematics teachers' curriculum includes problem solving and visualisation, although only further probing and research can confirm or reject this assumption.

Literature is replete with international coverage on the topic of practicing teachers modelling problem solving using visualisation. However, there is limited research on pre-service teachers in this field of study. Therefore, by conducting this study, I hope to extend research about Science and Mathematics teacher education in the national and international domain.

1.3 Problem statement

This study is based on an exploration of pre-service Science and Mathematics teachers' use of visualisation in a problem solving context in a South African university.

1.4 Research questions

There are three main research questions that guide the study. Each question has sub-questions to further explore and answer the main questions.

What do pre-service Science and Mathematics teachers understand by problem solving within a visualisation context?

What is pre-service Science and Mathematics teachers' understanding of problem solving?

What is pre-service Science and Mathematics teachers' understanding of visualisation?

What is pre-service Science and Mathematics teachers' understanding of the link between problem solving and visualisation?

Why do pre-service Science and Mathematics teachers who use visualisation strategies choose these strategies when teaching problem solving?

What visualisation strategies do pre-service Science and Mathematics teachers use when solving problems?

Why do these pre-service Science and Mathematics teachers use the visualisation strategies that they use?

How do pre-service Science and Mathematics teachers include the use of visualisation strategies when preparing their lessons?

How do pre-service Science and Mathematics teachers plan the use of visualisation strategies into problem solving?

What strategies would they use in the planning of teaching problem solving using visualisation strategies?

1.5 Definition of terms

The first common phrase used in this study is 'Pre-service Science and Mathematics teachers'. It is used largely because the case study focuses on the understanding of pre-service Science and Mathematics teachers in relation to visualisation, problem solving, and the link between the two. The pre-service teachers that are referred to in the study are teachers in training towards a degree in the field of Science and/or Mathematics education. All pre-service teachers who took part in the study were registered for modules in both Science and Mathematics, which incorporated problem solving, among other modules.

The second term used throughout the study is 'Problem solving'. Problem solving in this study refers to a Science or Mathematics statement which requires a solution, usually by using one or more operations (Collins, 2009). Pre-service Science and Mathematics teachers were studied to obtain their understanding of problem solving, not in isolation, but in relation to visualisation.

Visualisation is the third commonly used term in the study. Visualisation is a noun and the act of visualising refers to forming a mental picture (Collins, 2009). In relation to the study, pre-service Science and Mathematics teachers' understanding and use of visualisation is examined. There are two ways in which they could visualise a concept, firstly, by doing it internally (in their minds) by looking at information given in terms of a Science or Mathematics problem to be solved. Secondly, they could visualise the solution to a problem by using mediating artifacts or drawing diagrams to help make sense of a problem statement and thereby solve it.

1.6 Layout of chapters

This thesis comprises five chapters. The second chapter, the literature review, includes research syntheses and meta-analyses, which are critical evaluations of material that has already been published in the field of problem solving, visualisation, and pre-service teachers. Chapter 3 discusses the methodology used in this study. It describes in detail how the study was conducted, including conceptual and operational definitions of the variables used in the study. In the third chapter, all tools used to obtain data are also described and their appropriateness to generate the data is justified. Chapter 4 outlines the results and analysis of the research as well as results that run counter to expectations. Chapter 5 highlights the conclusions and implications of this study. This chapter concludes the study and includes limitations of the research as well as implications for further research, major conclusions and recommendations for future research.

1.7 Conclusion

A brief synopsis of the rationale for this study, including motivations based on school curricular requirements and teacher education have been described. The argument for the connection between problem solving and visualisation has been presented. The main and sub-research questions, as well as definitions of terms which are the focus of this study have been outlined.

The next chapter includes a literature study, which was reviewed and which is related to the objectives of this study.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

The following literature review is based on current national and international literature about the pre-service Mathematics and Science teachers modelling problem solving through the use of visualisation.

2.2 The South African context: Learner performance in Science and Mathematics

One of the main reasons for the initiation of this research is the poor performance of South African learners in Mathematics and Science. The performance will be discussed at two levels, high school and primary school. Tables 1 and 2 give a detailed analysis of South African high school learners' performance in the form of a Schools Subject Report. This report presents the performance of each school in the country in terms of performance in key subjects. For the purposes of the current study, the focus will be on the Mathematics and Science subjects in KwaZulu-Natal (KZN) (respectively highlighted on Page 8).

The comprehensive report presents all high schools in each of the nine provinces; it shows the number of Grade 12 learners who wrote the following common subjects: Mathematics, physical Science s, accounting, economics, geography, history, and English First Additional Language, as well the percentage pass rate for each.

Table 1 displays the overall learner performance per province, per subject in 2012. The following analysis will focus on the Science and Mathematics subjects in KZN. The purpose of this is to provide a detailed understanding of the need for the current study.

Table 1: Grade 12 Learners' Performance by Province in Selected Subjects in 2012

	Accounting		Agricultural Sciences		Business Studies		Economics		English (FAL)		Geography		History		Life Sciences		Mathematical Literacy		Mathematics		Physical Sciences	
	Wrote	% Achieved 30% and above	Wrote	% Achieved 30% and above	Wrote	% Achieved 30% and above	Wrote	% Achieved 30% and above	Wrote	% Achieved 30% and above	Wrote	% Achieved 30% and above	Wrote	% Achieved 30% and above	Wrote	% Achieved 30% and above	Wrote	% Achieved 30% and above	Wrote	% Achieved 30% and above	Wrote	% Achieved 30% and above
Eastern Cape	17 273	61.5	14 652	78.6	22 972	72.0	18 396	63.7	58 064	95.1	25 794	66.4	13 573	77.8	39 007	63.5	29 925	78.8	37 038	38.1	25 603	50.4
Free State	6 892	71.7	1 269	86.6	10 181	70.2	6 205	76.0	21 372	98.6	8 107	81.6	3 047	86.2	11 709	82.4	14 870	94.9	9 512	64.8	8 487	68.6
Gauteng	23 626	68.7	885	80.5	40 278	85.0	24 531	75.6	58 276	99.4	33 829	83.9	18 215	92.7	42 129	79.7	56 718	95.8	33 682	71.0	29 001	70.1
KwaZulu-Natal	40 642	66.4	17 823	76.8	51 804	85.8	34 116	81.1	102 280	97.7	52 757	75.9	23 394	89.6	68 750	67.0	65 313	80.3	63 168	48.1	45 951	58.3
Limpopo	17 984	62.3	24 293	70.6	21 353	61.8	22 139	70.5	75 630	98.5	38 899	74.1	11 392	78.0	48 169	65.0	42 516	83.8	35 044	52.4	30 975	59.9
Mpumalanga	11 251	54.9	13 428	65.9	18 215	68.1	13 448	59.8	44 080	96.3	20 418	69.7	5 665	71.3	24 672	65.3	29 248	84.9	18 835	53.1	16 493	63.2
North West	5 810	62.6	4 418	78.4	8 911	81.0	5 448	82.4	24 394	99.7	13 794	79.1	4 551	89.8	15 270	68.5	16 939	94.5	10 344	59.6	9 225	62.5
Northern Cape	2 001	74.2	691	71.6	3 508	70.4	1 568	74.2	8 025	99.4	4 050	74.5	2 598	84.6	5 425	61.9	6 265	91.7	2 864	54.9	2 202	60.1
Western Cape	9 499	75.9	689	87.1	18 285	74.2	8 518	69.4	27 918	98.9	16 087	80.0	12 054	91.3	23 281	78.6	29 547	94.5	15 387	73.5	11 257	70.9
National	134 978	65.6	78 148	73.7	195 507	77.4	134 369	72.9	420 039	97.9	213 735	75.8	94 489	86.0	278 412	69.5	291 341	87.4	225 874	54.0	179 194	61.3

Source: Department of Education (2012, p. 6).

Table 1 presents learners' results based on eleven subjects taken at Grade 12 level in 2012. It compares the results of eleven subjects in nine provinces. The table also displays the total of each subject under 'national'. In addition to this, it shows the number of learners who wrote a particular subject and the number of those who achieved 30% and above for each subject. The 30% mark is important because for most subjects at a secondary school level, 30% is the pass mark. The reason for the inclusion of this table is revealed in the yellow highlighted section which shows the low Science and Mathematics pass rate in KwaZulu-Natal. This is meaningful to this study because it is these poor results that form the basis for the need for the study.

In the subject Life Science s, KZN was ranked the fifth lowest percentage out of the nine provinces. In Physical Science s, KZN ranked the seventh lowest percentage out of the nine provinces. In Mathematical Literacy, KZN was ranked last. In Mathematics, KZN was ranked as the second lowest. In the Science s, KZN had the highest number of students taking both physical and biological Science s compared to any other province. The same is true for both Mathematics subjects.

Table 2 shows learners' performance in the eleven most popular subjects from 2009 to 2012 nationally. The most popular subjects are: Mathematics, Physical Science, Accounting, Economics, Geography, History, and English First Additional Language.

Table 2: Summary of Grade 11 Learners' National Performance in Key Subjects from 2009 to 2012

Subjects	2009			2010			2011			2012		
	Wrote	Achieved at 30% & above	% achieved	Wrote	Achieved at 30% & above	% achieved	Wrote	Achieved at 30% & above	% achieved	Wrote	Achieved at 30% & above	% achieved
Accounting	174 347	107 156	61.5	160 991	101 093	62.8	137 903	84 972	61.6	134 978	88 508	65.6
Agricultural Sciences	90 136	46 597	51.7	85 523	53 573	62.6	77 719	55 404	71.3	78 148	57 571	73.7
Business Studies	206 553	148 469	71.9	200 795	142 742	71.1	187 677	147 559	78.6	195 507	151 237	77.4
Economics	153 522	109 955	71.6	147 289	110 824	75.2	133 358	85 411	64.0	134 369	97 842	72.8
English First Additional Language	486 755	451 428	92.7	449 080	424 392	94.5	414 480	398 740	96.2	420 039	410 999	97.8
Geography	215 120	155 481	72.3	209 854	145 187	69.2	199 248	139 405	70.0	213 735	162 046	75.8
History	90 054	65 025	72.2	87 675	66 428	75.8	85 928	65 239	75.9	94 489	81 265	86.0
Life Sciences	298 663	195 652	65.5	285 496	212 895	74.6	264 819	193 946	73.2	278 412	193 593	69.5
Mathematical Literacy	277 677	207 326	74.7	280 836	241 576	86.0	275 380	236 548	85.9	291 341	254 611	87.4
Mathematics	290 407	133 505	46.0	263 034	124 749	47.4	224 635	104 033	46.3	225 874	121 970	54.0
Physical Sciences	220 882	81 356	36.8	205 364	98 260	47.8	180 585	96 441	53.4	179 194	109 918	61.3

Source: Department of Education (2012, p.5).

Table 2 presents a list of significant Grade 11 subjects ranging from 2009 to 2012. These results are based on the number of learners nationally. The Science and mathematical subjects are highlighted due to the significance thereof to this study. As with Table 1, the low results are noticeable.

Overall, in the Science s there has been a decrease in the number of Grade 11 learners taking these subjects. In 2009, the total for the Science s was 519 545, and in 2012 it was 327 787. That is a difference of 146 758 learners opting not to take up a subject in the field of Science. The question that it invokes is simply, why?

There has being a decrease in the number of Grade 11 learners studying subjects in the Mathematics field. In 2009, the total number of Grade 11 learners for Mathematics was 845 761 and in 2012 the total was 367 581. That means that 478 180 South African learners decided not to take Mathematics. In addition to this fact, the number of learners taking mathematical literacy increased from 2009 to

2012 and the number of learners taking Mathematics decreased in the same time frame. A reason could be that mathematical literacy is simpler or easier than Mathematics.

Table 2 reveals changes in the pass rates. The pass rate in Mathematical Literacy steadily increased, whereas Mathematics shows a staggered increase. Between 2009 and 2010, there was a 1.4% increase, and between 2011 and 2012 there was a 7.7% decrease in the Mathematics pass rate. This difference in the Mathematics pass rates could be attributed to different reasons for the different provinces and different schools. This data indicates general poor performance of learners in KZN in the fields of Mathematics and the Science s. Given this context, I have elected to explore pre-service teachers' understanding of visualisation within a problem solving context in the fields of Science and Mathematics education is examined.

Two worldwide evaluations, the TIMSS (Trends in International Mathematics and Science Study) and ANA (Annual National Assessments), are discussed. TIMSS is designed to help countries all over the world to evaluate student learning in Mathematics and Science. It collects educational achievement statistics from Grade 4 and Grade 8 to provide data about trends in performance over time, together with extensive background information to address concerns about the quantity, quality, and content of instruction. The ANA has the same purpose but it is a study done nationally and focuses on literacy and Mathematics/numeracy for all grades. ANA 2012 was a massive undertaking with over seven million learners nationally writing this test, and the results were as follows.

The national average performance for Grades 1, 2, 4 and 5 in numeracy in 2012 as compared to the results in 2011 is shown in Table 3.

Table 3: The national average performance for Grades 1, 2, 4 and 5 in numeracy in 2011 and 2012.

Grade	2011	2012
Grade 1	63%.	68%
Grade 2	55%	57%
Grade 4	28%	37%
Grade 5	28%	30%

Table 3 shows an increase in the pass rate for numeracy from 2011 to 2012.

The different levels of performance with learners performing above 50% for literacy/language show that:

- In Grade 3, 57% of learners achieved above 50% in Literacy, compared to 31% in 2011.
- In Grade 6, 39% (Home Language) and 24% (First Additional Language) of learners achieved above 50% in language, as compared to 15% in 2011.

In terms of the different levels of performance for numeracy/Mathematics in 2012:

- In Grade 3, 36% of learners achieved above 50%, compared to 17% in 2011.
- In Grade 6, 11% of learners achieved above 50%, compared to 12% in 2011.

The following achievement was recorded in other grades for numeracy/Mathematics in 2012:

- In Grade 1, 77% of learners achieved above 50%.
- In Grade 2, 68% of learners achieved above 50%.
- In Grade 4, 26% of learners achieved above 50%.
- In Grade 5, 16% of learners achieved above 50%.
- In Grade 9, 2% of learners achieved above 50%.

The results show an overall improvement in Mathematics from 2011 to 2012. It also shows that learners who fared better in literacy performed better in Mathematics. Language is an issue in South Africa because many learners are not English first language speakers. In the TIMSS study, a similar finding was made regarding the issue of language. The study showed that in countries where a large proportion of learners are from homes where the language of the test is not spoken at home, the Mathematics and Science scores are generally lower. In South Africa, 26% of learners reported that they "almost always or always" speak the same language at home and at school, while 9% reported that they "never" speak the language of the test at home. The scores of those who speak the language of the test at home were higher than for those who do not speak the language of the test at home (Reddy, 2012). With ANA, districts now have a standard source of information to determine which schools are most urgently in need of support (Motshekga, 2012).

On a larger scale, TIMSS Mathematics and Science in brief, ranks South Africa in the bottom three in the world. Further analysis into TIMSS findings show that from 2002 to 2012 there has been an improvement in both Mathematics and Science and this improvement is equivalent to raising the standard by one and half grade levels (Reddy, 2012). It seems that the greatest improvement was among learners who can be described as "the most disadvantaged" and who scored the lowest initially (Reddy, 2012). Alternatively, top performing South African learners and schools were not globally competitive. A comparison of the curriculum for these two disciplines revealed that the Revised National Curriculum Statements that guided instruction and learning of Mathematics and Science at schools during 2002 and 2011 covered more than 90% of the TIMSS assessment framework on which the learners were tested (Reddy, 2012).

In terms of qualifications, 60% of Mathematics learners and 53% of Science learners were taught by qualified teachers. Internationally, 87% of Mathematics learners and 90% of Science learners are taught by teachers who have completed a degree. South African schools are no stranger to the issue of unqualified teachers in schools and, as this report shows, it is a contributing factor to learners' performance (Reddy, 2012).

In considering the data and its analyses, it is clear that Science and Mathematics need more attention, not only in KZN, but also nationally. One of the assumptions of the current research is that teachers' competence in using visualisation strategies, along with problem solving skills, would aid in increasing the overall pass rate of learners.

2.3 Visualisation

Visualisation is an aspect that will be discussed in relation to problem solving, as well as on its own. Therefore, it is important to have a clear understanding of the two terms. The Collins English dictionary gives the root word of 'visualisation' which is 'visual' and it comes from Late Latin 'vīsuālis', from Latin 'vīsus' sight, from 'vidēre' to see. The Collins English Dictionary also gives the following definition for the verb 'visualise' and definitions for the noun 'visualisation' respectively, "1. To form a mental image of (something incapable of being viewed or not at that moment visible); 2. The act or an instance of visualising. 3. A technique involving focusing on positive mental images in order to achieve a particular goal," (Collins, 2009, p. 1820). Spreading from visualisation are many examples which can be termed 'visualisation strategies', 'visualisation tools' or, 'visualisation techniques'. Essentially, all three terms mentioned above can and will be used interchangeably in the current study.

Yilmaz, Argun, and Keskin (2009) define visualisation by looking at other authors' definitions: "[v]isualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings" (p. 131). However, this definition seems to lack the essence of what visualisation is and that is, "visualisation supplies depth and meaning to understanding," (Zimmerman & Cunningham, 1991 as cited in Elliot, 2001, p. 45). This study will be influenced by the view that visualisation enables greater depth of understanding of concepts.

The 'depth and understanding' as mentioned by Zimmerman and Cunningham (1991) can be acquired via visualisation strategies, e.g.: "drawing a diagram, [m]ake a list or chart" (Wheatley, 1995, p. 4); models and experiments, as explained by Gilbert and Justi (2010); concept mapping, as the basic principle of mind mapping techniques is to visualise concepts and link the concepts appropriately as explained by Fischer, Bruhn, Gräsel, and Mandl (2002); interactive and computer-based technologies; dynamic modelling tools; simulations; and networked multimedia environments (Angeli & Valanides, 2004), to name a few. These visualisation strategies are also used as teaching tools, as outlined in the Curriculum and Assessments Policy Statement (CAPS) (Department of Education, 2011a). The latest version of South Africa's curriculum is called the National Curriculum Statement (NCS), which comprises the Curriculum and Assessment Policy Statement (CAPS) and an additional two policies. CAPS is discipline specific and, for the purposes of the current research, I will examine CAPS for Mathematics and Natural Science s. CAPS for both Mathematics and Natural Science highlights the importance of making use of visualisation and the importance of learners being problem solvers (Department of Education, 2011a; Department of Education, 2011b). Visualisation and problem solving will be discussed in terms of the Mathematics and Natural Science CAPS respectively.

One of the outcomes in the Mathematics CAPS declares that learners should be able to "communicate effectively using visual, symbolic and/or language skills in various modes" (DoE, 2011a, p.5). The 'various modes' or approaches mentioned here (that learners should be able to do) link in with multiple representations. However, at this stage one should keep in mind that one critical aspect of basic education is that learners are able to use different visual styles. The different 'visual styles' may be summed up as visual strategies. Therefore, simply put, the Mathematics CAPS aims for learners to be able to use visualisation strategies. Similarly, one of the outcomes in the Natural Science CAPS declares that learners should be able to "communicate effectively using visual, symbolic and/or language skills in various modes" (DoE, 2011b, p. 5). Also, learners should be able to "communicat[e] – using written, oral, visual, graphic and other forms of communication to make information available to other people" (DoE, 2011b, p. 11). The Natural Science s CAPS adds a new perspective to what learners should be able to do and that is to use visualisation to make information available to others. This is interesting as it does not merely say that learners should know how to use visualisation, but also be able to communicate to others by using it.

2.4 Problem solving

Apart from the importance of visualisation, both the Mathematics and Natural Science CAPS also highlight the importance of learners being capable of becoming problem solvers. Problem solving has varied meanings, for the purpose of this study, problem solving will be defined as the activities that one engages in when attempting to find a solution to a question with no apparent path to the solution (Lewis & Smith, 1993; Wheatley, 1995; Hobden, 2002).

The Natural Science s CAPS has many problem solving outcomes that it envisages for learners. It states that learners should be able to identify problems and issues, as well as being able to articulate the needs and wants of people in society. Furthermore, CAPS informs that learners should be able to raise questions, but not just questions, but articulate relevant questions about problems, issues, and natural phenomena. In being able to ask questions, it is further hoped that learners will become capable of identifying, solving problems and make decisions using critical and creative thinking. Along with being critical thinkers, they should demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation. At this juncture, one cannot help but wonder how all this is possible within the reality of the classroom. From anecdotal evidence, it has been observed that more time is spent on administrative work rather than covering all the above aspects meaningfully. The list goes on to mention that learners should be able to complete investigations, analyse problems and use practical processes and skills in evaluating solutions. To summarise the importance of problem solving in the Natural Science s CAPS is for learners to be able to identify and solve problems creatively (DoE, 2011b, p. 11).

Problem solving seems to be a regular feature in the Natural Science s CAPS and it seems that these proposed outcomes are not being met, as articulated by Hobden (2002) when he states that "[r]ecent research has shown that students and teachers have a limited and narrow understanding of what it means to solve problems" (p. 1). This gives the impression that learners and teachers cannot become problem solvers if they do not understand what it means to solve problems.

2.5 The link between visualisation and problem solving

In the Mathematics CAPS, problem solving is emphasised in its outcomes where it states that learners should be able to "identify and solve problems and make decisions using critical and creative thinking; …demonstrate an understanding of the world as a set of related systems by recognizing that problem solving contexts do not exist in isolation" (DoE, 2011a, p. 5). The importance of problem solving and visualisation can be seen in both the MMathematics and Natural Science CAPS, and therefore it is a teacher's responsibility to make every effort to adhere to the recommendations made in the policy documents. In saying this, the current study suggests that problem solving can be made more comprehensible by using visualisation.

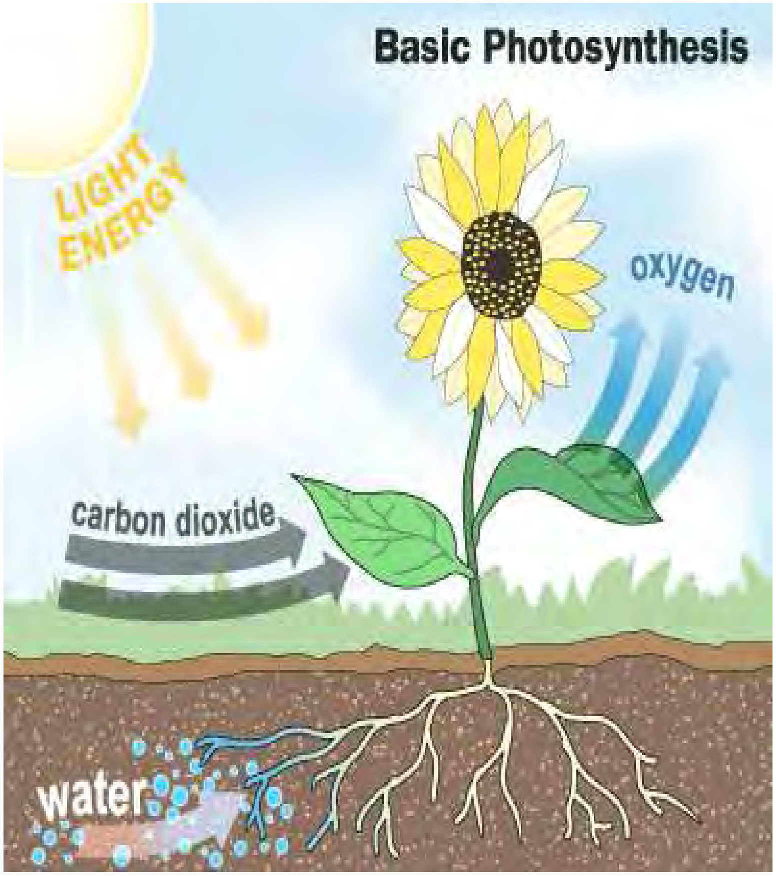
With the definition and problem solving presented, it is important to establish a link between the two. Ho (2009) confirms the link between problem solving and visualisation in stating that "[v]isualisation can be a powerful cognitive tool in problem solving [… the] ability to reason visually is increasingly important in the information age. Thus, the role that visualisation plays in students' mathematical thinking and problem–solving experiences has become more significant" (p. 1). Although Ho (2009) highlights visualisation as being important when solving problems, Elliot (2001) discusses the "ability to apply and interchange between both visual and non-visual methods in problem solving [as it is] particularly advantageous for students" (p. 46). In keeping with the value of using visual and non-visual methods, Angeli and Valandies (2004) bring another dimension to the discussion. They discuss the effects of text-only versus text-and-visual usage when solving problems, "Results showed that the text-and-visual group outperformed the text-only group" (Angeli & Valanides, 2004, p. 23). Their finding shows the value of collaboration between visualisation and text when solving problems.

Miller (1987), as cited in Gilbert (2004), adds to this view of visualisation being valuable in solving problems from the side of models used in Science . He states that "[t]here is a general agreement that visualisation is an importan[t] component in scientific achievement" (p. 123). For the above to be successful, teachers have to be able to convey the two aspects of visualisation and non-visualisation as one when teaching learners.

2.6 Visualisation and non-visualisation methods

Using different visualisation methods as well as non-visualisation methods to solve problems could make learners of different intelligences to understand a particular idea or concept. One such example can be seen in Table 4 where a visual and non-visual explanation is given about photosynthesis.

Table 4: Comparison of visualisation and non-visualisation explanation of Photosynthesis

Visualisation Example	Non-visualisation example
	In higher plants, photosynthesis involves chemical reactions in which the sun's energy is transferred along a series of oxidation and reduction events until it is stabilized in the chemical bonds of glucose. In the broadest sense, light energy converts carbon dioxide (CO_2) into chemical energy while water is split to release oxygen. Internal view of a chloroplast. Most photosynthesis occurs in the leaves of plants, although there may be photosynthetic stems, flowers, and fruits. At the cellular level, photosynthesis occurs inside organelles known as chloroplasts. Plants use photosynthetic pigments (e.g. chlorophyll) to capture the light energy which is ultimately converted into chemical energy in the form of sugars.

Source: Wilson (2008).

Table 4 presents the process of photosynthesis, which is explained in two ways, firstly by visual methods and secondly, by non-visual methods. It highlights an important point that, 'a picture speaks a hundred words'.

The concept of photosynthesis is a simple one to the experienced teacher, but to learners who are exposed to it for the first time, they may appreciate a diagram (visual) representation as opposed to a page of notes explaining the process. Alternatively, some learners may prefer a fact sheet (worksheet) to a picture. The point of this example is that different learners learn in different ways: some learn better with pictures, some learn better through words, and some learn better through music, for example. This means that they have varied intelligences.

2.7 Multiple intelligences

These different intelligences are termed 'multiple intelligences'. The theory of multiple intelligences (MI) was introduced in 1983 by a Harvard University professor, Howard Gardener, Gouws(2007). He categorised people's thinking and learning into eight categories:

Table 5: Eight categories of people's thinking and learning

Intelligence	Explanation
Linguistic intelligence	Word smart, the ability to learn languages.
Logical-mathematical intelligence	Number/reasoning smart, the ability to detect patterns, reason deductively and think logically.
Spatial intelligence	Picture smart, the potential to recognise and use the patterns.
Bodily-Kinesthetic intelligence	Body smart, the ability to use mental abilities to coordinate bodily movements.

Intelligence	Explanation
Musical intelligence	Music smart, the capacity to recognise and compose musical pitches, tones, and rhythms.
Interpersonal intelligence	People smart, the capacity to understand the intentions, motivations and desires of other people.
Intrapersonal intelligence	Self-smart, entails the capacity to understand oneself, to appreciate one's feelings, fears and motivations.
Naturalist intelligence	Nature smart, enables human beings to recognise, categorise and draw upon certain features of the environment (not on Gardeners official list of intelligences).
Existential intelligence	Thinking smart, empirical evidence is sparse (not on Gardeners official list of intelligences).

Source: Adapted from Armstrong (2008), and Gouws (2007).

Table 5 displays the nine types of intelligences and an explanation of each. These categories were developed further by other researchers in the MI field. Such additions were: Spiritual intelligence and Moral intelligence (Smith, 2007).

In terms of MI and the South African Outcomes Based Education (OBE) classes, research shows that "[t]he introduction of OBE in South African schools demands that educators would start teaching beyond the 'traditional' intelligences, and more specifically beyond those falling within the linguistic and logical-mathematical parameter" (Gouws, 2007, p. 67). In moving away from the 'traditional' intelligences, we will most likely attract the attention of more learners. However, the effectiveness of using the MI theory depends on the facilitator in charge, namely, the teachers. This is supported by one of the findings of a case study done by Pienaar, Nieman, and Kamper (2011). Their study involved implementing a teaching approach based on the multiple intelligences theory in a South African school. Their findings concluded that "properly trained and committed teachers, a school environment conducive to change and constant monitoring are important prerequisites for the implementation of a successful MI approach to teaching" (Pienaar, Nieman & Kamper, 2011, p. 284). They noted that it is not only the teacher who is crucial in making MI theory effective, but the whole education system. Gouws (2007) reiterates the importance of teachers being properly trained when he states that, "[i]t is a cause for concern that not all educators in South African schools are adequately trained and/or feel confident enough to incorporate MI in the classroom" (p. 72). This is one of the reasons that in the current study, I decided to work with pre-service teachers.

2.8 Rationale for this study

The current study focuses on pre-service Mathematics and Science teachers because, first, from anecdotal evidence, teachers do not model adequate problem solving skills using visualisation, and second, literature alludes to the need for this focus. Therefore, it is hoped to go to the 'root' of the problem by focusing on pre-service teachers before they are influenced by teachers already in service.

Crespo (2003) found that pre-service teachers shy away from problems that are higher order and pedagogical order as they themselves are not well informed (in terms of content knowledge) or equipped (in terms of skills, such as visualisation) to understand such problems, let alone teach it to their learners. As a result of this, Crespo (2003) explains that pre-service teachers use simple problems and aid learners to a point where learners are unable to learn from their errors, and this results in missed learning opportunities. She points out an example from her study where the pre-service teacher used easy problems: "she [the pre-service teacher] thought of ways to prevent […] difficulties from arising. This and similar pre-service reactions […] suggest a concern with sparing pupils from becoming confused, getting a wrong answer, and spending too much time on problems" (Crespo, 2003, p. 253).

Crespo's findings suggest many changes need to be made in the way that pre-service teachers teach and understand problem solving. Gilbert (2004) also discusses the changes that need to be made and includes possible changes in pre-service teachers' education curriculum. He notes that “the demands that these requirements place on teacher education are considerable and […] need to be explicitly addressed” (p. 127). This is an issue that the current study will also be considering. The current study focuses solely on pre service teachers and, if one is to adhere to requirements in the CAPS documents then “it is vital that both teachers and students see the role of visualisation clearly and use it to help them in their problem solving process” (Ho, 2009, p. 3).

Although teachers should recognise the role of visualisation in problem solving, they should not neglect the text aspect. The ‘text’ part of problem solving refers to the use of formulae and algorithms to solve a problem. Angeli and Valanides (2004) conducted a study in which they examined the effects of text-only and text-and-visual instructional materials during problem solving. They worked with two groups who had to solve problems using either text-only techniques or text-and-visual techniques. Their results showed that the “text-and-visual group outperformed the text-only group” (Angeli & Valanides, 2004, p. 23). Based on this finding it can be assumed that visualisation would be more effective if used in conjunction with text (algorithms).

In A similar study, Lowrie and Kay (2001) found that “visualisation strategies on occasion are efficient” (p. 253). Angeli and Valanides (2004) go on to add that the success of the learners does not only rely on whether they chose text-only or text-and-visual strategies, but also on their cognitive abilities. This brings to light that it is not merely the strategy used in reaching the learner that counts, but it also depends on their own intellectual ability. “Modelling tools are, perhaps, the most intellectually demanding technologies that ‘enhance the cognitive powers of human beings during thinking, problem solving, and learning’” (Jonassen & Reeves, as cited in Angeli & Valanides, 2004, p. 23). Therefore, it is possible that having a variety of visualisation strategies in teaching problem solving will empower learners who have low cognitive abilities by developing their problem solving skills.

Similarly, the study by Lowrie and Kay (2001) also adds a new perspective to the visual versus non-visual debate and this involves the difficulty of a problem having an influence on which strategy they choose. They found that “nonvisual methods were used in less difficult situations” (p. 253). One can deduce from the above that task complexity influences task representation. However, it can also be argued that a visual representation of a task is not usually the most efficient or effective way to solve a specific Mathematicsproblem (Aspinwall, Shaw, & Presmeg, 1997; Lowrie, 1996 as cited in Lowrie & Kay, 2001).

> Nonvisual methods, based on particular algorithms or strategies, are usually more concise. The notion that non-visual methods provide an effective and efficient way of representing word problems has implications for teachers in elementary schools. An individual's ability to represent a problem analytically may have been predetermined by the availability of a repertoire of nonvisual problem solving strategies. That list of strategies would include, but not be restricted to a student's capacity to represent problems algebraically (Lowrie & Kay, 2001, p. 254).

Ultimately the choice lies with the learners and what they choose will be determined by their cognitive ability and the type of intelligence which dominates their process of understanding.

2.9 Multiple representations

In catering to learners' different cognitive abilities, it is my opinion that multiple representations (MRs) can be used in relation to the teaching and learning of problem solving. MRs is a set of varied ways used to make sense of a concept or a process in any subject. It is another term for visualisation techniques. Examples of MRs vary from subject to subject (Ozel, Capraro, & Yetkiner, 2008). For the purposes of the current study, examples in the areas of Mathematics and Science will be mentioned. Such examples include: web-based learning tools; the Virtual geo-board instrument,

> [This] instrument employs several research protocols to improve the quality of the data collected and to ensure that data gathered is not the result of computer or internet based resources. The instrument uses screen capture every 5 seconds to record the cursor position and the work area content. The purpose of this screen capture is to provide precise information about student progress both as they make successful progress in the solution process as well as capturing any false starts so we have a complete accounting of each attempt (Ozel, Capraro, & Yetkiner, 2008, pp. 4, 5).

Examples of MRs are :graphics, puzzle pieces, charts, models, diagrams, gestures, software code, videos, projectors, simulation programs, pictures, number lines, and graphic calculators to name a few. The list is rather extensive and to prove this, a periodic table of visualisation methods can be added on, as seen in Figure 1.

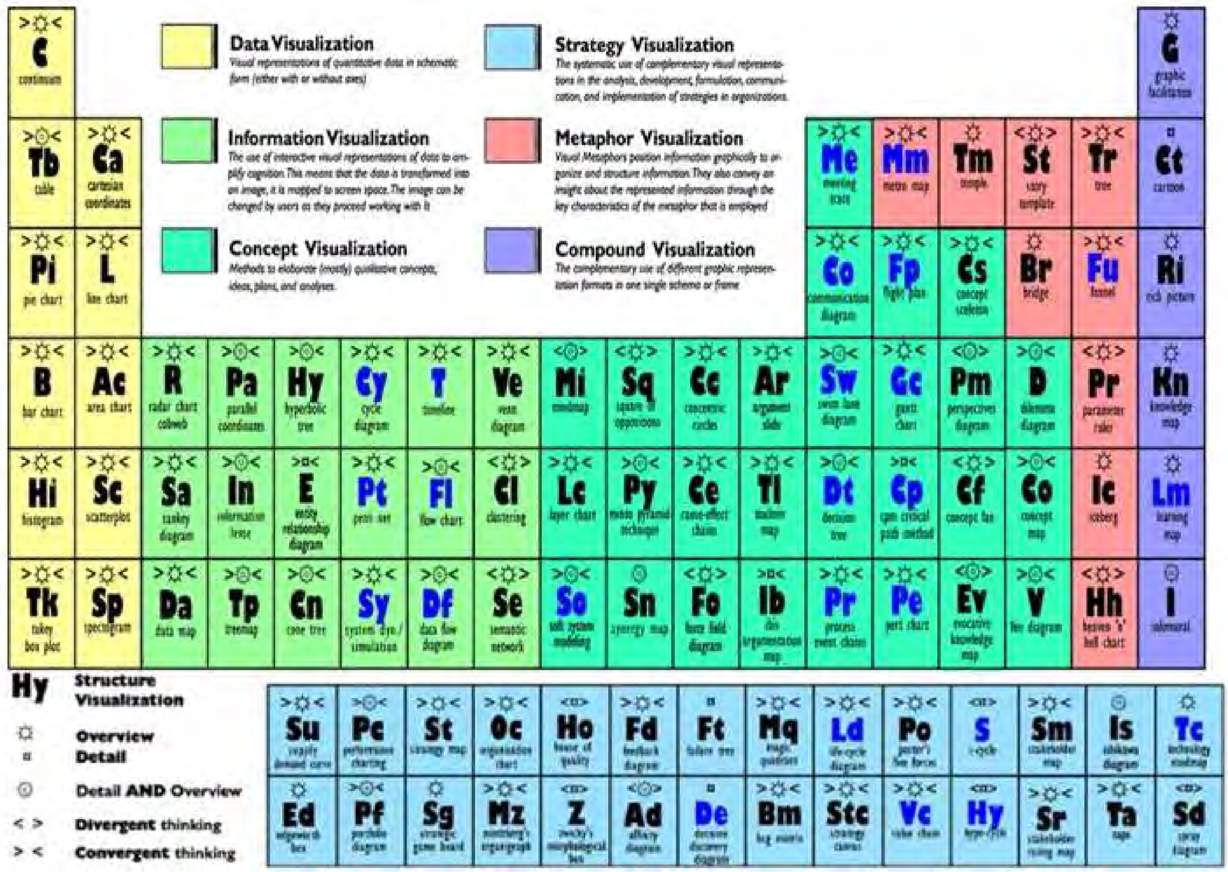


Figure 1: Periodic Table of Visualisation Methods (Lengler & Eppler, 2007, p. 5)

Figure 1 is meant to be similar in nature to the periodic table of elements, however the elements are replaced by types of visualisation methods. Like the periodic table of the chemical elements, the periodic table of data visualisation methods categorises and explores the visualisation techniques using a taxonomy based on the nature of visualisation. The assortment of visualisation strategies are categorised into six categories, namely: Data Visualisation; Information Visualisation; Concept Visualisation; Strategy Visualisation; Metaphor Visualisation; and Compound Visualisation.

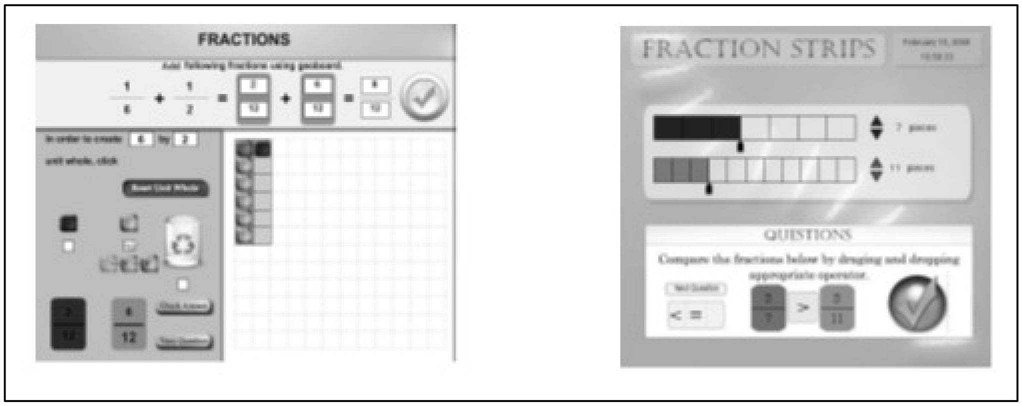


Figure 2: Screen shots of a Virtual Geo-board and Fraction Strips (Ozel et al., 2008, p. 4)

Figure 2 presents screen shots of a Virtual Geo-board and Fraction Strips respectively.

A Virtual fraction strip instrument, “contains many of the same features as the virtual geo-board with the visual variation of the screen layout” (Ozel et al., 2008, pp. 4, 5). The examples of MRs, especially by incorporating visual strategies in Figures 1 and 2, represent unconventional ways of teaching and learning Mathematics and Science.

> In the educational psychology literature, there is a substantial amount of evidence demonstrating that the use of multiple representations of learning content *(MRs) can significantly enhance student learning* [emphasis added] in complex domains, compared to learning with only a single representation. However, simply providing a learner with multiple representations (e.g., textual description plus graphic, or multiple graphical representations [MGRs]) does not necessarily result in flexible knowledge acquisition.It has been argued that learners must perform a number of cognitive tasks in order to benefit from MRs (Rau, Aleven, & Rummel, 2009, p. 1).

Rau et al. (2009) raise two important aspects of MRs: they can enhance learning but guidance is needed if these MRs are to be effective. By extension, only if MRs are used correctly, can their advantages be fully effective. One of the many advantages is that by “using multiple representations to demonstrate the same concepts helps not only to develop a better conceptual understanding but also strengthens one’s ability to solve problems” (Amato, 2004; Gagatsis & Elia, 2004 as cited in Rau et al., 2009, p.2). A challenging problem solving question would require a good conceptual understanding and therefore MRs seems to cater to this requirement.

In addition to MRs developing conceptual understanding, it is also argued that “MERs [Multiple External Representations] support different ideas and processes; that MERs constrain interpretations and; [and] that MERs promote a deeper understanding of the domain” (Ainsworth, Bibby, & Wood, 1997, p. 94). The ‘domain’ in this case (in linking it to the current study) would be the area of solving problems. One of the main advantages proposed for the use of MERs is that by using combinations of representations, we can exploit their different properties to aid learning (Ainsworth, Bibby, & Wood, 1997).

A relatively new acronym that is linked to the advantages of MRs is Virtual Manipulatives and Whiteboard (VMW) (Hwang, Su, Huang and Dong, 2009). VMW, or Virtual Manipulatives and Whiteboard, involve the use of smart boards and whiteboards. Figure 3 is an example of a VMW.

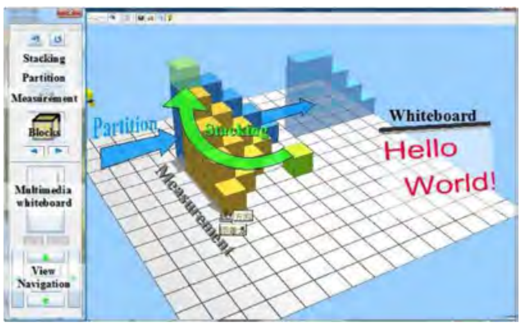


Figure 3: The manipulation tools in VMW system (Hwang, Su, Huang and Dong, 2009, p. 229)

“The purpose of the VMW (in Figure 3, for example) is to promote a multi-representative construction model with which users can easily organize their thinking manually and symbolically to solve geometry problems” (Hwang, Su, Huang, & Dong, 2009, p. 229). VMW’s allow learners to interact with what appears to be abstract problems.

Ozel et al. (2008) also add to the debate of the advantages of MRs. The authors state the following:

> The use of multiple mathematical representations promotes students’ understanding of mathematical concepts. Research indicates that positive gains in understanding of mathematical topics appear in cases when multiple modes of mathematical representations are used effectively. Technology integration facilitates multiple modes of representations which improves transitions from concrete manipulation to abstract thinking, and provides a foundation for continued learning (p.2).

The authors here, as with Rau et al. (2008), emphasise the link between MRs and the development of conceptual understanding, and that with conceptual understanding comes effective learning.

Özmantar, Akkoç, Bingölbali, and Ergene (2010) also acknowledge the link MRs have on effective learning. They argue for the use of MRs by maintaining that:

> Use of multiple representations (MRs) is important as they can potentially create conditions for effective learning and as they lead to deeper levels of understanding of the subject. Research on MRs show that unless the links between and among the MRs are stressed, students experience difficulties in connecting the MRs by themselves. However teachers do not explicitly focus on the links in their instructions (Özmantar, Akkoç, Bingölbali, Demir & Ergene, 2010, p.19).

Özmantar et al. (2010) urge teachers to understand the link between the different MRs, and mention that learners will make better use of the MRs available if they understand the link between these MRs. In making better use of the MRs, learners stand to develop deeper conceptual understanding (Özmantar et al. (2010). MRs are made up of various technologies, as mentioned earlier on in this section. Nonetheless, in looking at the technologies available, one must be careful not to stray from the fact that Mathematics and Science *have* a numerical part to them. However, Friedlander and Tabach (2001) reason that the numerical part has its flaws, and that MRs can aid in filling the inadequacies in the purely numerical approach.

> A numerical approach may not be very effective in providing a general picture and as a result, some important aspects or solutions of a problem may be missed…*The graphical representation* is effective in providing a clear picture of a real valued function of a real variable. Graphs are intuitive and particularly appealing to students who like a visual approach […] *The algebraic representation* is concise, general, and effective in the presentation of patterns and mathematical models and hence, it is a powerful tool. The manipulation of algebraic objects is sometimes the only method of justifying or proving general statements […] *The verbal representation* is usually used in posing a problem, and is also needed in the final interpretation of the results obtained in the solution process. The verbal presentation of a problem creates a natural environment for understanding its context and for communicating its solution. Verbal reasoning can also be a powerful tool for solving

problems and can facilitate the presentation and application of general patterns" (Friedlander & Tabach, 2001, pp.174-175).

Friedlander and Tabach (2001) make mention of three examples of MRs, which are: graphical representation; algebraic representation; and verbal representation. They discuss all the above in terms of their advantages. As much as they view the numerical approach as sometimes inefficient, they also point out that the use of "numbers is important in acquiring a first understanding of a problem, and in the investigation of particular cases" (p.147). Friedlander and Tabach (2001) deem all of these to be effective tools in helping learners learn better. They introduce a 'new' MR in the form of verbal representation, one that they see as a crucial example of MRs. The figure below sums up the advantages of using MRs to aid learners in solving problems, as documented by Özmantar.

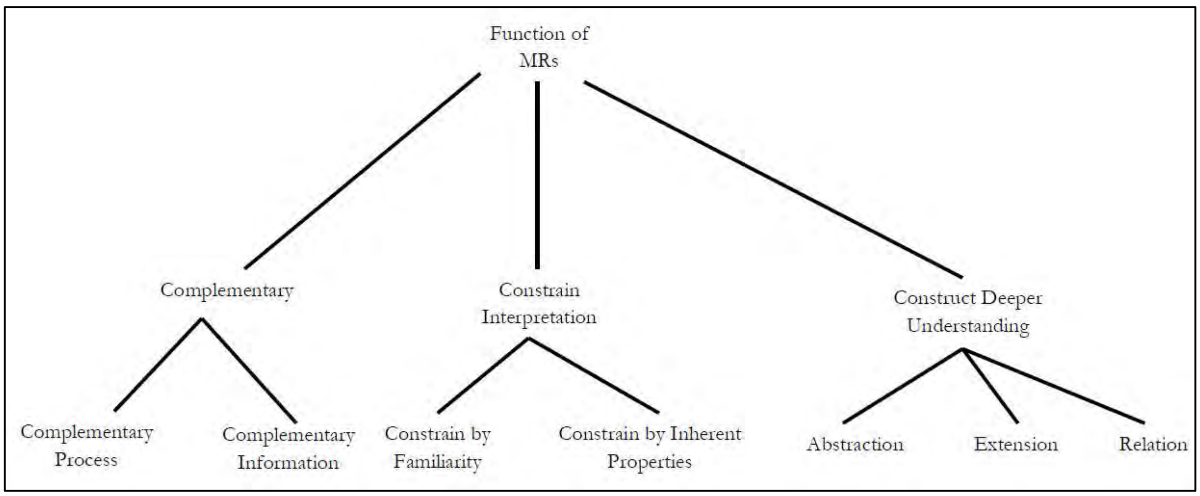


Figure 4: A functional taxonomy of multiple representations (Ainsworth, 1999, p.134 as cited in Özmantar, 2010, p. 21)

The last example of MRs that will be discussed is the use of models. Models "are means of visualisation; they serve mainly pedagogical purposes and in some cases may have a heuristic function" (de Chadarevian, 2004, p. 288). Models were not always accepted as a teaching and learning tool. De Chadarevian (2004) describes the introduction and inclusion of models into Mathematics and Science. She claims that Felix Klein is an iconic figure in the development of models in the late 1860's and was at the driving end of creating some, and collecting other, models to create an initial collection of models in Mathematics and Science. Descriptive geometry is what was encouraged in the teaching of engineering in the late 1800s, this is where the need for models came into play. The term that was used was 'Anschauung', which "means the perception by the sense of sight and simultaneously the mental conception of an object" (Meyers Konversations – Lexikon, 1984, as cited in de Chadarevian, 2004, p. 292). Essentially, it describes what we know today as visualisation techniques.

2.10 Models as an example of multiple representations

De Chadarevian (2004) focuses on the use of models and states that the first model seems to have been created in France, which resulted from the meeting of Mathematics and technology. Models were later used in museums "to 'teach people' by stimulating their imagination and emotions through visual impressions […] biological group exhibits would convey to the naïve visitor a quantity of information about casual relationships that would otherwise be mystifying" (de Chadarevian, 2004, p. 317). This concept of 'teaching people' is what can be taken into the teaching and learning of Mathematics and Science in our classrooms.

Further advantages of models are discussed in the writings of Copeland (1979). One of the focuses of his book is looking at the methods of teaching Mathematics courses in teacher education. He examined how students learn and, by extension, how one could help students better understand the content taught. He asserted that memory and Mathematics can be summarised as stating that students learn better if given models to work with and those students remember a model better than written words. Copeland (1979) stresses Piaget's view that to memorise something is not merely to make a copy, but rather to show understanding. He [Piaget] explains that there are three levels of memory, the first which is being; the second which is initiative, and the third which is operational. It is the third level that shows students' understanding.

It seems as though most authors are in favour of the use of MRs and, after much reading, a few disadvantages of MRs were found. The discussion on the disadvantages of MRs will touch on the following aspects: verbal language; numerical representation; graphical representation; the use of algebraic symbols; and finally, the cost factor.

In terms of the use of verbal language, Friedlander and Tabach (2001) contend that verbal language "can be ambiguous and elicit irrelevant or misleading associations; it is less universal and its dependence on personal style can be an obstacle in mathematical communication. Moreover, if symbols are avoided, verbal communication or reasoning may be a less powerful mathematical tool than more formal approaches" (p. 174). de Chadarevian (2004) supports this view that language can be misleading when she explains models in terms of 'Anschuung'. She states that "[a]nschuung is vivid, clear, and objective, while words and letters are empty and dead" (p. 292). One has to wonder whether verbal language does not enhance the effectiveness of a problem solving method, and if it is eliminated, how will it affect the student holistically? The authors also deliberate over the example of numerical representation and make note of its lack of generality as a disadvantage. "A numerical approach may not be very effective in providing a general picture and as a result, some important aspects or solutions of a problem may be missed" (Friedlander & Tabach, 2001, p. 174). Perhaps in the early grades, solving problems through numerical representation can be minimised, but the point of education and in turn, problem solving, is to help learners grow cognitively, and by oversimplifying everything, it is a genuine concern that the quality of their education will be below par.

Another (and perhaps controversial) disadvantage of MRs is found in terms of graphical representation. The literature reveals a lack of accuracy in using the MRs, "[G]raphical representation may lack the required accuracy, is influenced by external factors (such as scaling) and frequently presents only a section of the problem's domain or range" (Friedlander & Tabach, 2001, p. 174). One may argue that the way in which the graphical representations are presented to students is the responsibility of the teacher. Therefore, it is the teacher's responsibility to have a good understanding of the different types of MR and the knowledge of how to best present it to students. The example of algebraic symbols is brought under scrutiny. Friedlander and Tabach (2001) claim that "an exclusive use of algebraic symbols (at *any* stage of learning) may blur or obstruct the mathematical meaning or nature of the represented objects, and cause difficulties in some students' interpretation of their results" (p. 174). In saying this, some students may still prefer an 'all algebraic' way of solving problems.

Finally, there is one other challenge related to the use of MRs in the teaching and learning process, which is cost factor, which was not found in the literature. The previous examples of MRs such as models, software codes, videos, projectors, and simulation programs are expensive. Most South African schools cannot afford these aids. The teacher can be trained to expose learners to a variety of MRs because different learners possess different cognitive ability and therefore would prefer one type of visualisation strategy over another.

2.11 Theoretical and conceptual frameworks

2.11.1 Suitability of the Metacognitive Theoretical Framework

The interpretivist paradigm underpinned this study philosophically. The framework used to develop and interpret the study is the metacognition theory. Metacognition refers to higher order thinking, which involves control over the cognitive processes engaged in learning (Livingston, 1997). Livingston explains metacognition as the 'process engaged in learning' to be achieved through activities such as "planning how to approach a given learning task, monitoring comprehension, and evaluating progress toward the completion of a task are metacognitive in nature" (p.1). In this study, pre-service teachers reflected on how they planned their lessons, and reflected on why they used visualisation and other methods in the way that they did in order to solve problems.

John Flavell originally conceived the term metacognition in the late 1970s and defined it as "cognition about cognitive phenomena" (Flavell, 1979, p. 906). Lai (2011) simplifies this meaning as thinking about thinking. This study included reflection on thinking and trying to understand how pre-service teachers think about their use of visualisation within a problem solving context. In addition to this, metacognition refers to the ability to be aware of and monitor one's learning processes (Schwartz, 2010). During the completion of the task sheets, pre-service teachers had to solve problems and explain why they had solved the problem the way that they did. In this way, pre-service teachers were made to think about their own learning process. Metacognitive theory is apposite because it can enable teachers to explain their cognitive performance or plan effectively.Therefore, teacher training curricula should include ways to help construct metacognitive awareness (Schwartz, 2010).

Rahim and Siddo (2009) conducted a study on the use of visualisation for learning and teaching Mathematics, which is comparable to this study because it involves visualisation and how pre-service teachers learn from it, as well as how they would teach using visualisation. In Rahim and Siddo's (2009) study, they used a metacognitive theoretical framework to establish the effects of visualisation on learning (gaining knowledge) and teaching. Metacognition consists of knowledge and regulation. Metacognitive knowledge includes knowledge about oneself as a learner and the factors that might impact performance (declarative knowledge), knowledge about strategies (procedural knowledge), and knowledge about when and why to use certain strategies (conditional knowledge). The three types of knowledge that metacognition involves also form the basis of this study. Declarative knowledge is questioned in research question one, which asks what pre-service teaches' understanding is of visualisation and problem solving. Procedural and conditional knowledge is questioned via the task sheets where pre-service teachers had to explain why they used the strategies that they used to solve the problems. Both procedural and conditional knowledge are related to the lesson plan, which pre-service teachers were required to complete and include an explanation of how they planned the use of visualisation strategies in the context of problem solving. Therefore, the theoretical framework of metacognition seems most suitable for this study.

Metacognitive regulation is the observing of one's cognition and includes three skills, namely, planning activities, consciousness of comprehension and task performance (monitoring), and evaluation of the efficacy of monitoring processes and strategies (Lai, 2011; Schraw, 1998 as cited in Schwartz, 2010; Schraw & Moshman, 1995; Veenman, Hout-Wolters & Afflerbach, 2005; van Kraayenoord, 2010). These three essential metacognitive regulation skills explain the rudimentary and overall order in which this study was completed.

2.12 Paradigm

Research can be described as a systematic investigation of sources that leads to data collection, data analysis, and conclusions being drawn (Burns, 1997; Mertens, 2005, as cited in Mackenzie & Kripe, 2006). In order for research to be systematic, paradigms and theoretical frameworks are used. A paradigm indicates how a researcher thinks about the development of knowledge in making decisions and carrying out research. It helps the researcher to conduct the study in an effective manner (Williams, 2011). Depending on the type of research being done, different paradigms may be used. The paradigm and theoretical framework that were used in this study will be discussed respectively. Each will be reviewed in terms of its purpose and suitability to the study.

2.12.1 Suitability of the interpretivist social Science paradigm to this research study

The choice of paradigm one decides on for a study indicates to the reader the philosophical intention, motivation, and expectation for the study (Mackenzie & Kripe, 2006; Wahyuni, 2012). Paradigms are useful because, according to Gorland (2014), they allow for "creating a foundation which most scientists accept, (and) spares effort that might otherwise be exerted in constantly re-examining assumptions and can instead seek progress in fields which are less well understood" (p.1). Collinson (2012) and Wahyuni (2012) explain the fundamentals of what makes up a paradigm. The fundamentals are discussed via three characteristics, ontology (how one sees reality), epistemology (how one knows something), methodology (how one would go about finding out about something).

This study is underpinned by the interpretivist social Science paradigm. This paradigm, which is also known as the constructivist paradigm, is used to understand and describe meaningful social actions. Cohen and Manion (1994) indicate that the interpretivist paradigm should be used in research that aims to understand "the world of human experience" (p.36). The interpretivist paradigm is suitable because this study proposes to 'understand' pre-service teachers' use of visualisation in a problem solving context. The 'social action' also refers to the interaction between the pre-service teachers and the researcher throughout the study, gained by analysing their responses to the face-to-face interviews. In addition to this, the 'social action' the study aims to describe is that between pre-service teachers and their learners in terms of how they would teach visualisation in a problem solving context. This is included in research question three, which questions how pre-service teachers include the use of visualisation strategies when planning their lessons. This data relied on pre-service teachers completing lesson plans in relation to the use of visualisation within a problem solving context. All the data collected in the study relied on the responses of the participating pre-service teachers. Creswell (2003) explains that an interpretivist researcher relies on the "participant's views of the situation being studied" (p.8). This indicates that an interpretivist paradigm is apposite to this study.

Mackenzie and Knipe (2006, p.3) expand the meaning of the interpretivist paradigm by mentioning that an interpretivist study "is most likely to rely on qualitative data collection methods and analysis". 'Relying on qualitative methods' is an apt way to describe the methods used in this study. The data collected from the pre-service teachers were their understandings and interpretations, as well as their solutions to problems, followed by reasons for their solutions. All tools used to collect data in the study were aimed to be of a nature to gather qualitative data. Typically, qualitative research uses tools such as open ended questionnaires, interview schedules, reflective journals, and methods such as observations (Cohen & Manion, 1994). These tools and methods were used in this research, which further supports the argument for the suitability of the use of the interpretivist paradigm in this study (Wahyuni 2012). Although the study is qualitative in nature, percentages (quantitative) are used when representing certain responses to provide a deeper understanding of the pre-service teachers' experiences as these relate to the research questions.

2.13 Conclusion

The literature reveals that South African learners, especially in grades 11 and 12, produce poor results in both Science and Mathematics. It also explains the importance of problem solving and the use of visualisation in schools Science and Mathematics curricula. The literature reviewed revealed how visualisation via multiple representations and catering to learners' multiple intelligence can assist learners to perform better in problem solving and help them to achieve better results in Science and Mathematics. The next chapter will focus on a discussion of the research design and methodology used in the study.

CHAPTER 3 METHODS AND METHODOLOGY

3.1 Introduction

In the previous chapter, research in the areas of problem solving and visualisation in relation to pre-service Science and Mathematics teachers was discussed.

This chapter will highlight the methods and methodology of the current research. The primary difference between method and methodology is that research methods are the approaches by which you conduct research into a subject or a topic. Alternatively, research methodology explains the methods by which you may proceed with your research, Cram(2013). Cram (2013) goes on to elaborate on the technical differences by stating that research methods involve conducting experiments, tests, surveys and the like. Research methodology involves the learning of the various techniques that can be used in the conducting of research and in the conducting of tests, experiments, surveys and critical studies. Cram's (2013) differentiation between method and methodology may be summarised as methods being the strategies used to gather data, whereas methodology involves the research which underpins the use of the strategies and tools to generate data.

The present research was framed within the interpretivist paradigm with reference being made, to a large extent, to qualitative data collection and analysis, Mackenzie & Knipe(2006). This paradigm proved to be suitable because it relies on qualitative methods such as open-ended questionnaires, interview schedules, reflective journals, and methods such as observations (Cohen & Manion, 1994). The theoretical framework that underpins this study is metacognition. Metacognition also explains that a study should include planning activities, consciousness of comprehension and task performance, and evaluation of the efficacy of monitoring processes and strategies (Lai, 2011). The use of lesson plans, open-ended questionnaires, and interview schedules incorporated these aspects of metacognition.

To discuss the method and methodology of the current study, five categories will be elaborated on. They are: Research Design, Research Settings and Sample, Research Instruments and Procedures, Validity and Reliability, Ethics, and finally, a summary of the chapter.

3.2 Research Design

Research design refers to the structure of an enquiry; it is a logical matter rather than a logistical one. Research design is not related to any particular method of collecting data or any particular type of data, be it quantitative or qualitative. Qualitative methods are used with the intention of understanding people's views and their interpretations. For this reason this study uses qualitative methods to gather data. At its core, qualitative research focuses on the meanings, traits and defining characteristics of events, people, interactions, settings, cultures and experience. As one leading advocate of qualitative methods has explained, "Quality refers to the what, how, when, and where of a thing – its essence and ambience. Qualitative research thus refers to the meanings; concepts; definitions; characteristics; metaphors; symbols; and descriptions of things" (Berg, 2007, p. 3). Different methods are used to gather the information mentioned above. Such methods include interviews, observations, focus groups, open-ended questionnaires, and video taping of interviews. From the methods mentioned here, this study made use of interviews, which were audio recorded; observations; and an open-ended questionnaire. The data that was gathered from the above mentioned methods was analysed by looking for patterns and commonalities between them. The methods of analysing qualitative data involve reading and re-reading the collected data. The process of gathering the data will affect the analysis of the findings. The same is true when working with quantitative methods in research.

No mention is made of amount or quantity in dealing with qualitative research. This is because the numerical descriptions of subjects or objects and their relationships is not the focus of qualitative research, it is the focus of quantitative methods. Quantitative research, which is one of the two primary approaches to study the conduct of social Science research, is used when the researcher wants to use statistics to support a theory. This research approach is an objective, formal, systematic process in which the findings are presented numerically. It describes, tests, and examines cause-and-effect relationships using a deductive process. The focus is on using specific definitions and carefully operationalising what particular concepts and variables mean (Charoenruk, 2007; Tewksbury, 2009). Quantitative research methods in the social Science s are broader, and often address issues at the macro societal level, where qualitative methods are impractical to use. This research tool is administered to a large number of people. A tool that may be used in collecting quantitative data is a closed ended questionnaire. The data that is collected from such a method would be analysed by looking at patterns and common categories, coding, and then representing the data in the form of diagrams and tables.

The qualitative research design which was adopted in this study was used to address the following research questions:

1. What do pre-service Science and Mathematics teachers understand by problem solving within a visualisation context?

2. Why do pre-service Science and Mathematics teachers who use visualisation strategies choose these strategies when teaching problem solving?

3. How do pre-service Science and Mathematics teachers include the use of visualisation strategies when preparing their lessons?

To help answer these research questions meaningfully, sub questions to each research question were established.

3.3 Research Settings and Sample

The study was carried out at a South African university, which commits itself to excellence in research-led teaching and learning. This institution for teaching and learning is host to pre-service teachers. The requirements for these pre-service teachers' entrance to the institute are typically that they must be Grade 12 graduates who have obtained a Bachelor's degree pass and have come through a selection process that gives privilege to academic achievement. These students are considered to be above average school performers and have come from a diverse school context, including those that have experienced rural and urban schooling. The majority of these students would have made teaching their first or second choice of study and therefore have a keen interest in education. It is for this reason that the teacher training campus was utilised as the research setting for this study.

3.3.1 The pilot study

The pilot study was conducted with a group of ten pre-service Science and Mathematics teachers. These pre-service teachers were third year students and registered for, among others, two modules namely, Natural Science Method 2 and Mathematics Method 2. They were given the questionnaires, a Science problem solving worksheet, a Mathematics problem solving worksheet, and a lesson plan to complete. In addition to responding to the data generation instruments, the respondents were asked to comment on how the tools could be refined. The comments that respondents gave about the questionnaire related to the lack of space for pre-service teachers' contact details. Respondents also suggested that in Section B, Question 2.1, which read "Have you worked with visualisation strategies?" be extended by adding the prompts "When" and "How". Once the change was made, the question then read, "Have you worked with visualisation strategies? When and how did you do this?" In the same section of the questionnaire, 2.8 read, "Which skills have you used?" This referred to the visualisation strategies that they had used. Respondents suggested changing this question to read, "Which skills have you used most frequently?" In terms of the task sheets, respondents commented that the space given was not ample. Thereafter, more space was inserted for solving the problem and their explanations. With regard to the lesson plan, respondents wrote their lesson plan on the title page provided. It was then observed that it should have been indicated on the page that it was a cover page and should accompany their lesson plan.

All the comments that the pilot group submitted were taken into consideration. Thereafter, my supervisors, as well as a peer, proofread the instruments.

3.3.2 Sample

A class of eighty pre-service Science and Mathematics teachers in their third year were identified, which became the potential sample group for this study. Purposive sampling methods were used, as well as convenience sampling methods in selecting this sample group. Both these sampling methods will be explained respectively in relation to this study. Sampling is the use of a sub-group of the population to represent the whole population; the population of this research study was pre-service Science and Mathematics teachers.

3.3.2.1 Appropriateness of purposive sampling in relation to this study

Purposive sampling is also referred to as non-probability sampling (Laerd, 2012; Trochim, 2006; Tongco, 2007). Purposive or non-probability sampling means that a sample group is selected intentionally and is by no means random. This is understood to mean that sampling is done with a purpose in mind. It signifies sampling done as a series of strategic choices about with whom, where and how to do one's research (Palys, 2009). Trochim (2006) explains this further by describing that "[t]he purposive sampling technique, also called judgment sampling, is the deliberate choice of an informant due to the qualities the informant possesses (p.1)".

This research study is a qualitative, exploratory one, therefore purposive sampling is an appropriate method. This view is supported by Laerd (2012) when he states that "purposive sampling is particularly useful in exploratory qualitative research" (p.3). Trochim (2006) and Palys (2009) likewise suggest that purposive sampling be used in qualitative research. This further validates the decision to use purposive sampling in this study. Crossman (2014) explains that if research utilises purposive sampling, then the subjects need to be selected based on some characteristic/s. In this research, the sample group was selected based on the characteristic of their option and year of study. In this case, subjects were chosen who were pre-service Science and Mathematics teachers in their third year of study. The other reason was to analyse how these pre-service teachers used visualisation in a problem solving context. Being an exploratory study of pre-service teachers, the data that was generated was made up of experiences, emotions, and opinions. This is fitting in purposive sampling, as explained by Trochim (2006). The results generated were therefore not used to make statistical generalisations but, as Laerd (2012) mentions, it can however help in making logical generalisations.

Purposive sampling has subcategories. The subcategory that this study falls under is heterogeneity sampling. This method refers to the inclusion of all opinions or views (Trochim, 2006).

Three benefits of using purposive sampling are explained by Tongco (2007) and Laerd (2012). Firstly, purposive sampling, when used appropriately, is more efficient than random sampling because the random member of a community may not be as knowledgeable and observant as someone in the field of study. Secondly, this method is especially useful when there are not enough funds and other resources (Laerd, 2012). Thirdly, one of the major benefits of purposive sampling is the wide range of sampling techniques and methods that can be used across such qualitative research designs. This is noted in this study where qualitative data tools such a questionnaire, task sheets, observation, reflective journal, lesson plan and interviews are utilised.

3.3.2.2 Convenience sampling in relation to this study

Convenience sampling is another type of sampling method. Convenience sampling was used in this research study in conjunction with purposive sampling. Therefore, the sampling method used is referred to as purposive convenience sampling.

This study followed the convenience sampling method because the members of this particular population were selected because the researcher had easy to access them and they were willing and available to be participants (Oliver, 2002; Laerd, 2012). Having access to the pre-service teachers and their availability was important for me as the researcher because it meant that I could meet with them readily. The advantage of this type of sampling was the availability and the rapidity with which data could be gathered. In some instances, however, the respondents did not submit completed instruments back on time. This limitation is explained by Price (2013) when she states that convenience samples are "rarely convenient to draw, but they are referred to this way to distinguish them from random samples" (p.2).

An additional advantage of using convenience sampling is that the relative cost and time required to carry out a convenience sample are small in comparison to probability sampling techniques. This enables one to achieve the sample size required in a relatively fast and inexpensive way (Laerd, 2012). This proved to be true in this study.

In conclusion, the use of both purposive and convenience sampling proved to be both fitting and effective in selecting subjects to research the topic of this study, which involved pre-service Science and Mathematicsteachers. The combination of sampling methods served this study well.

3.4 Research Instruments and Procedures

The instruments were designed to respond to the research questions. Each data collection tool was developed based on data needed to answer each research question. This also correlated with the stages of data collection in the study.

Table 6: Outline of this study

Stage of study and date	Tool used	Data gathered	Pertaining to research question (RQ)	Sub-research questions
Stage 1	Questionnaire.	1. Biographical data 2. Pre-service teachers' understanding of problem solving, visualisation, the link between problem solving and visualisation, the visualisation strategies they were aware of and used, and their views on the importance of the use of visualisation strategies in a problem solving context as well as the importance of teaching it.	(RQ1)What do pre-service Science and Mathematics teachers understand by problem solving within a visualisation context?	1. What is pre-service Science and Mathematics teachers' understanding of problem solving? 2. What is pre-service teachers' understanding of visualisation? 3. What is pre-service teachers' understanding of the link between problem solving and visualisation?
Stage 2	Task sheets (one on Science problems and one on Mathematics problems).	1. How pre-service teachers solve Science problems as compared to Mathematics problems. 2. What strategies they use to solve problems and why they choose these strategies.	(RQ2) Why do pre-service Science and Mathematics teachers who use visualisation strategies choose these strategies when teaching problem solving?	1. What visualisation strategies do pre-service Science and Mathematics teachers use when solving problems? 2. Why do these pre-service teachers use the

Stage of study and date	Tool used	Data gathered	Pertaining to research question (RQ)	Sub-research questions
		3. How they explain the use of their strategy (the assumption here was that if they could explain how they solved a problem, then they could explain it to their learners).		visualisation strategies that they use?
Stage 3	Lesson plan.	How pre-service teachers use visualisation strategies to teach problem solving.	(RQ3) How do pre-service Science and Mathematics teachers include the use of visualisation strategies when preparing their lessons?	1. How are pre-service teachers taught problem solving using visualisation? 2. How do pre-service teachers plan the use of visualisation strategies into problem solving? 3. What strategies would they use?
Stage 4	Interview 1.	Clarification of responses from pre-service teachers.	-	Every interview was different based on what needed clarification from each pre-service teacher.
Stage 5	Interview 2.	Clarification of responses from pre-service teachers.	-	Every interview was different based on what needed clarification from each pre-service teacher.

Table 6 outlines the stages of the research study, what data generation instrument was used, what data it was developed to gather, and which research question each tool was developed to answer.

3.4.1 Stage one of data collection

This stage made use of the questionnaire which was aimed at answering research question one, which was: What do pre-service Science and Mathematics teachers understand by problem solving within a visualisation context? This research question was answered via three sub-questions. Firstly, what is pre-service Science and Mathematics teachers' understanding of problem solving? Secondly, what is pre-service teachers' understanding of visualisation? Thirdly, what is pre-service teachers' understanding of the link between problem solving and visualisation?

This information was important in comparing pre-service Science and Mathematics teachers' understanding of problem solving and visualisation, as well as the relationship between the two. The source of this information was the pre-service Science and Mathematics teachers. They comprised eighty pre-service teachers from the combined class of pre-service Science and Mathematics teachers. Of the eighty pre-service teachers who the questionnaire was administered to, twelve completed and returned it. This low return rate is acknowledged by Dey (1997) when she states that "[n]ational data show[s] a continuing decline in the willingness of people to respond to surveys. This trend is troubling given the central role that survey research plays in collecting data for institutional research purposes" (p.215). Dey's article which was written in 1997 is crucial because it indicates that low response rates is not a new issue and it is an issue that is perpetuated from year to year. Later research by Sivo, Saunders, Chang, and Jiang, (2006), and Holbrook, Krosnick, and Pfent, (2007) also discuss the issue of low response rates in their research, as well as the decrease in response rates from year to year.

Response rates are strongly affected by the method of data collection. This study used qualitative data collection methods, which relied directly on pre-service teachers completing them. In a study titled 'Data Analysis Australia: Response Rates'by Henstridge and his team (2014***)***, it was revealed that self-completion surveys (such as this study's questionnaire) often have lower response rates than telephone interviews or face-to-face interviews. This is because the respondent is left to return the survey. It also requires a greater degree of effort on the part of the respondents, because they are requiredf to fill in the questionnaire themselves without assistance and then return it. However, the reason that the questionnaire was selected as the first tool is because it could be administered to many pre-service teachers at once. It did serve its purpose, although the response rate was poor.

To avoid this low response rate, Sivo et al. (2006) and Holbrook et al. (2007) suggest that in general, the more interaction between the potential respondent and the researcher, the higher the response rate. De Heer (1999), and Curtin et al. (2000, 2005, cited in Holbrook et al., 2007) suggest a way to combat this trend of low response rates. They suggest that researchers include longer field periods to collect data, increased numbers of call attempts to respondents, sending advance letters to serve as reminders, and offering incentives. In relating their suggestion to this study, I made telephone calls and sent emails reminding respondents about submitting completed instruments. Only twelve of the eighty questionnaires were received, nine of the twelve task sheets, and five of the nine lesson plans were returned.

Roszkowski and Bean (1990) suggest the length of a questionnaire affects the response rate. They explain that the longer the questionnaire, the lower the return rate. It seems that respondents become bored and/or uninterested in completing questionnaire that have no impact on their class grade or if no incentive is offered. Dillman (2007, as cited in Miller & Dumford, 2014), supports this study's explanation for the low response rate for the questionnaire. They state that completing open-ended response options requires a greater amount of time and mental effort from respondents than most close-ended questions require. Another issue with the questionnaire, as stated by Miller and Dumford (2014), is that open-ended questions have much higher rates of low to nonresponse than other types of survey items. This study's questionnaire was based on open-ended questions. In retrospect, a few closed ended questions should have been included in the questionnaire. In addition to this, they suggest placing the most important open-ended questions at the beginning of the questionnaire.

There are many issues around low response rates. However, high response rates do not necessarily ensure external validity (Miller and Dumford, 2014). Nevertheless, the researcher cannot be sure that the conditions of external validity are met when response rates are low. The poor response rate is particularly troublesome for descriptive studies such as this one because their usefulness lies in their capacity to generalise the findings to a population with high confidence. Such low response rates jeopardise the generalisation of findings (Sivo et al., 2006). However, in this study, statistical generalisation to the wider population was not intended.

In referring back to the questionnaire, information about the pre-service teachers' understanding of problem solving and visualisation were collected from the participant's written responses in section B of the questionnaire (refer to Appendix A) at the beginning of the study. Section A comprised pre-service teachers' biographical information as well as their contact details. The questionnaire was the most suitable instrument because it was less intimidating than a one-on-one interview (at this early stage of the study with the participants) and in turn minimised the Hawthorne Effect. This instrument allowed participants to openly give their understanding and still take time to think about their responses. Also, the questionnaire could be administered to a large amount of participants at once, thus saving time.

3.4.2 Stage two of data collection

At the second stage of data collection, two task sheets were used. One task sheet comprised Science problems and the other Mathematicsproblems for pre-service teachers to solve. The data collected was used to answer research question two, which was: Why do pre-service Science and Mathematics teachers who use visualisation strategies choose these strategies when teaching problem solving? This research question was answered via two sub-questions. Firstly, the goal was to find out what visualisation strategies pre-service Science and Mathematics teachers use when solving problems. Secondly, the goal was to find out know why these pre-service teachers use these visualisation strategies. The answers to these questions are important in order to know how many different visualisation strategies the participants had used. If a participant developed shallow or inadequate strategies, then it was hoped that an understanding could be obtained of why this was the case. The information about why they chose the strategies they that they used was useful in understanding why they do not use other strategies and why they prefer specific strategies. Although this study was not an action research project, it was hoped that some improvement would occur as these relate to pre-service teachers' understanding and use of visualisation through their participation in the study.

The task sheets required written responses from the participating pre-service teachers. Responses were generated when participants were asked to solve four Mathematics problems and three Science problems. In retrospect, there should have being an equal number of questions per task sheet. All participants responded to both sets of Science and Mathematics questions. They used diagrams and/or equations to solve the problems. This was followed by a written reason explaining why they used visualisation strategies, and their preference for specific strategies. The data was collected from the problem solving task sheets, which the participants completed (see Appendix B and Appendix C). The task sheets were suitable to collect information about which visualisation strategies these pre-service teachers used and why they used these strategies. The task sheet was a well suited instrument to use when looking for a response to this research question. The task sheets and questions allowed me to gain an immediate insight into the thinking which informed the way the participants solved the problems.

3.4.3 Stage three of data collection

In the third stage of the research process, pre-service teachers were requested to design a lesson plan focusing on teaching problem solving using visualisation strategies. The data collected from this instrument was used to provide insight into pre-service Science and Mathematics teachers' use of problem solving and visualisation strategies when preparing their lessons. The goal was to know how pre-service teachers taught problem solving using visualisation, as well as how they planned to use visualisation strategies in order to work with problem solving. The answers to the above questions gave me insight into how they would teach this (problem solving) to their learners. In addition, the research sought to find out how they would make visualisation flow or fit in with problem solving in a tangible way. The source of this information was a lesson plan that the participating pre-service Science and Mathematics teachers completed (see Appendix D). This data was collected once from the lesson plans that the participants submitted. Their written and/or drawn responses were then analysed.

The lesson plan was suitable because it gave me an idea of how these pre-service teachers intended to translate their understanding of visualisation and problem solving into practice. It gave them the opportunity to apply their theory and examples of problem solving within a visualisation context. A lesson plan allowed participants to, at their leisure, design a lesson. By applying their minds over a period of time it would have been possible for them to produce lesson plans that reflected how they would actually teach in class. This was a useful way of assessing the potential of each of the participants' use of visualisation strategies in their classrooms. This was also a useful means of establishing some form of triangulation after the questionnaires and interviews were completed. It was necessary to gauge the participants' practice of teaching and how they linked their lesson plans with what they wrote in their questionnaires, and their responses during their interviews.

The lesson plan was the final instrument used by the participants. The data which was generated from these lesson plans provided greater insight into how pre-service teachers would use visualisation and problem solving when they become practising teachers. This data was collected once during the research.

3.4.4 Stages four and five of the data collection process

Certain data from the instruments needed clarification. Some of the responses were either found to be ambiguous, unanswered, incoherent and/or incomplete hence the need for individual face-to-face interviews. Both interviews sought similar information that was solicited using the data generation tools, but the interviews took place at the end of the study. Its use at the end of the study was most effective because a rapport had been developed between myself and the participants. The second and final interview was also based on a need for further clarification of their responses

In the initial planning of the study, only one interview was scheduled but, due to the amount of clarification that was needed once the data analysis had begun, a second interview was clearly required. Both interview sessions were audio recorded with the permission of the pre-service teachers.

3.4.5 A summary of the data collection process

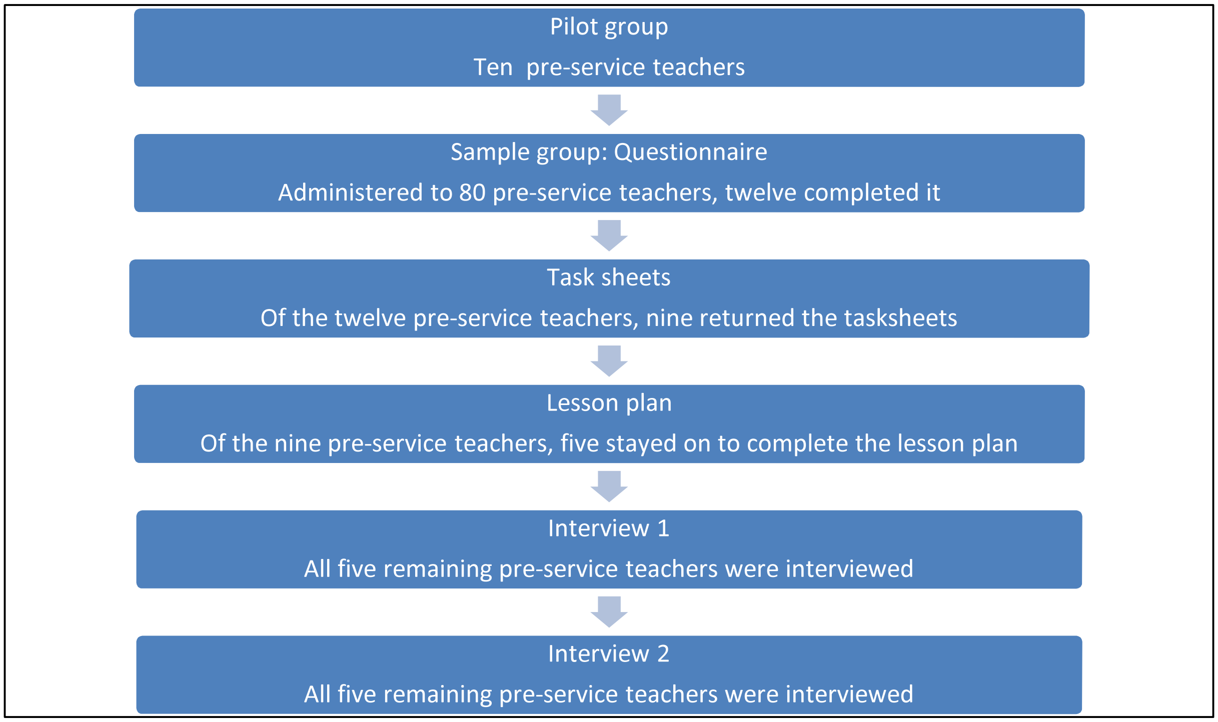


Figure 5: Summary of the data collection process followed in this research study

Figure 5 presents a summary of the data collection process of this research study. It comprises six levels at which the data was collected.

3.5 Validity and Reliability

"Validity determines whether the research truly measures that which it was intended to measure or how truthful the research results are" (Joppe, 2000 as cited in Golafshani, 2003, p.599). Validity is used to determine whether the instruments are relevant in attaining the required results for the research. In this research, validity was ensured by engaging a pilot group to test the instruments. Weak, ambiguous and redundant questions were highlighted and thereafter the instruments were refined. This was done to keep each instrument's purpose in focus. Each instrument was developed with a research question in mind. The questions in each instrument were developed by following the sub-questions of each research question.

During the interviews the participants were made to feel comfortable and, because they had met with the researcher many times before then, they felt at ease. Dates and times for each participant's interviews were set according to their timetables and availability. The interviews were recorded with permission from the participants.

Joppe (2000) explains reliability as "[t]he extent to which results are consistent over time and an accurate representation of the total population under study is referred to as reliability and if the results of a study can be reproduced under a similar methodology, then the research instrument is considered to be reliable" (Joppe, 2000, as cited in Golafshani, 2003, p.598). This study can be reproduced but the results may vary because contexts can vary. Also, the topics of problem solving and visualisation are both relatively common. Therefore, the reliability of the current research seems high. To increase the validity and reliability of this study, a triangulation of methods was used. The use of triangulation is significant because "triangulation strengthens a study by combining methods" (Patton, 2001, as cited in Golafshani, 2003, p.603). In order to triangulate, multiple data generation strategies were used. Participant responses in the first instrument, the questionnaire, were noted and compared in the responses to the second instrument, the task sheets, as well as the third instrument, the lesson plan. Most pre-service teachers showed development in their knowledge of problem solving and visualisation as they proceeded from one instrument to the next.

When a sample is representative, it becomes valid over the topic it represents, which means it possesses external validity. When a sample is measured suitably, it becomes valid for the sample, thus providing internal validity. Non-probability methods, which is what this study utilised, contributes more to internal validity than external validity (Tongco, 2007).

3.6 Ethics

Before the commencement of the study, all the participating pre-service teachers were briefed about the research and what their role in it was if they chose to take part. They were told that it was not compulsory for them to participate, but if they chose to do so they would have full autonomy and that they would be assured confidentiality of the information they supplied. They were also informed that they would remain anonymous and that codes would be used to identify their responses in the report.

Consent from the head of the School of Education, as well as well the College Research Ethics Committee to commence with the study was first acquired and thereafter the pre-service teachers were approached. Informed consent of the participants was also declared in writing (see Appendix E).

The pre-service teachers' names are not referred to anywhere in this study. They are referred to by codes which were allocated to them, for example, the pre-service teacher who submitted their questionnaire first is referred to as Pre-service teacher 1 throughout the study. Anonymity of the participants was maintained. During the analysis of the data, I was conscious of being unbiased and tried to interpret the responses as accurately as possible. In order to reduce researcher bias, I kept field notes and a reflective journal. These were kept as a reminder to be aware that I was working with data and that the data in itself is valuable, and on account of my personal preference, I as the researcher should not view one strategy as being superior to another.

3.7 Summary of Chapter

In this chapter, the method and methodology used to inform the research were discussed and explained. The choice of the interpretive paradigm and the qualitative approach was justified. A description of the instruments and the multiple methods used was presented. In order to enhance validity, a pilot study was done and instruments were thereafter refined. The questionnaire was submitted to eighty pre-service Science and Mathematics teachers. Twelve pre-service teachers returned the completed questionnaire. Subsequently, nine pre-service teachers submitted the task sheets and of the nine, five remained to complete the lesson plan. Two interviews were conducted to clarify the participants' responses.

In the next chapter, the empirical evidence gathered from the methods is presented and analysed.

CHAPTER 4 DATA ANALYSIS AND RESULTS

4.1 Introduction

The qualitative data which was generated using multiple methods are presented and analysed in this chapter. Details of metacognitive theory are presented in order to justify its use in analysing this work.This chapter explores the findings of this study, drawing on similarities and differences from other research that has been done in the field of pre-service Science and Mathematics teachers, as well as the use of visualisation in a problem solving context. The data was analysed in order of the emergent themes according to the specific research questions to which they relate.

Qualitative research also includes an analysis of respondents' cognition, called metacognitive regulation. As stated in the previous chapter the metacognitive regulation is the observing of one's cognition and includes three skills, namely (i) planning activities, (ii) consciousness of comprehension and task performance (monitoring), and (iii) evaluation of the efficacy of monitoring processes and strategies (Lai, 2011; Schraw, 1998, as cited in Schwartz, 2010; Schraw & Moshman, 1995; Veenman, Hout-Wolters, Afflerbach, 2005; van Kraayenoord, 2010). These three essential metacognitive regulation skills explain the skills that pre-service teachers practised as a result of being a part of this research study. The pre-service teachers planned lessons that included visualisation. Thereafter, the researcher evaluated the responses in relation to the research questions and also re-evaluated these to give herself a better understanding of the data gathered, thereby evaluating the pre-service teachers.

Schwartz (2010) and Lai (2011) describe three different types of metacognitive theories which are used to plan and learn cognitively. Firstly, there are tacit theories, these are unconscious frameworks that structure metacognitive knowledge. It is difficult to change because this theory alters as a person grows and learns. It is influenced by one's culture, peers, teachers, and experiences. Secondly, there are informal theories, these involve individuals who are aware of some of their beliefs and assumptions regarding phenomena, but have not yet constructed a theoretical structure that integrates and justifies these beliefs. People who would use this theory are aware of some theories, but lack the structure to organise their beliefs about knowledge. Thirdly, there are formal theories which are described as a systemised approach to learning something that has a specific form or structure that needs to be known. This type of theory is well structured as compared to the first and second theories mentioned above.

4.2 Data analysis and findings

Themes emerged from the data sets which were analysed. Each theme was verified by findings from the pre-service teachers' responses to the data generation tools. The classification of the themes according to the three research questions follows respectively.

4.2.1 Research question one

Research question one was: *What do pre-service Science and Mathematics teachers understand by problem solving within a visualisation context?* Three themes emerged from research question one, these are:

- Pre-service Science and Mathematics teachers' understanding of problem solving.
- Pre-service Science and Mathematics teachers' understanding of visualisation.
- Pre-service Science and Mathematics teachers' understanding of the link between problem solving and visualisation.

These three themes emerged from the questionnaire, which was the first tool used to generate data from the pre-service teachers. A total of eighty pre-service teachers were approached to participate in the study and only twelve pre-service teachers responded. The questionnaire was open-ended and comprised four sections. Section A included the participating pre-service teachers' biographical information. Section B (1) had questions based on problem solving. Section B (2) had questions based on visualisation. Section C had questions based on the link between problem solving and visualisation. Section D had a single question which asked pre-service teachers to present any additional comments with regard to problem solving within a visualisation context. The following analysis is of the pre-service teachers' responses to questions in the questionnaire.

- **Theme 1: Pre-service Science and Mathematics teachers' understanding of problem solving.**

This theme arose from responses to the questionnaire. A discussion on the responses of each related question follows:

- **Working with problem solving**

All the participating pre-service teachers indicated that they had worked with problem solving prior to answering the questionnaire. A reason for this could be that they were enrolled in modules which included problem solving in their respective curriculum. Their curriculum included problem solving, therefore it was anticipated that the pre-service teachers had the following skills at this stage in their training: problem identification, structuring of logical steps to solve a problem, decision making, verbal and non-verbal communication, and content knowledge of a second year university student.

- **Number of problems solved in a week**

Four out of twelve pre-service teachers indicated that they solved zero to three problems, and eight pre-service teachers solved four to ten problems during the preceding week. This finding is unusual because it was expected that all Science and Mathematics pre-service teachers should be engaged daily with problem solving, which is a requirement of the modules in the higher education curriculum. It was expected that students in the Science and Mathematics education cohort would be working with many Science and Mathematics modules. Inherent in each module are issues related to problem solving. Three possible reasons for the four students indicating that they did four problems or less are described: First is the possibility that they were not completing their exercises as instructed by the module lecturer. This may explain the poor results that the students obtained. Second, students did not receive sufficient exercises from the module lecturer and were expected to find and complete as many problems on their own as possible. These students may not have fulfilled this request and hence may have been disadvantaging themselves. Third is that these students did not understand the concept of problem solving.

- **Attitudes towards engaging in problem solving**

Seven pre-service teachers specified that they enjoyed solving problems and two indicated that they did not. Pre-service teachers who did enjoy solving problems reasoned that it enhanced their confidence when they worked out a problem correctly. However, some added that if they ‘get stuck’ it frustrated them. This is probably one of the reasons why teachers ought to encourage problem solving in classrooms. Through problem solving, it is expected that learners will develop their

Science and mathematical skills and this may lead them to attain some success. Also, this success may boost their confidence. The inability to solve a problem should evoke a greater yearning for the solution because it creates a challenge for the solver. Real life situations also sometimes appear as problems and there are occasions when these problems may seem insurmountable. It would be interesting to explore how Science and Mathematics teachers encourage their learners to persevere until they actually find the solution. Perhaps of greater importance to the education faculty are the two students who stated that they did not like problem solving, or the one student who said he sometimes enjoyed problem solving. Science and Mathematics classrooms abound with problem solving situations and it may be inconceivable that these students will become effective teachers of these subjects.

The latter three students believed that they could not relate problem solving to real life contexts and that it often took too long to find solutions. This may necessitate the revision of the curriculum at universities to include aspects of context so that these pre-service teachers can relate the problems they work with to real life situations. It is an important part of school Science and Mathematics and hence should be emphasised. The issue of taking too long to find solutions may indicate that these students did not really understand their Science or Mathematics. Without the basic *a priori* knowledge, problem solving would be difficult. Furthermore, a perusal of the curriculum content at the higher education institution reveals that there are no specific courses that offer students the possibility of learning about problem solving itself. Many modules contain problem solving, but it is not immediately evident whether pre-service teachers actually learn the theory behind solving problems. This may create challenges for these students when they become teachers in schools.

Pre-service teachers' attitudes to problem solving may be linked to whether or not they find problem solving useful. One pre-service teacher said that problem solving helps him to explore different ways of solving a problem. Six pre-service teachers stated that problem solving helps them to develop skills that include logical thinking, applying knowledge, cognitive development and scientific skills. It is important to note that most of the participating pre-service teachers stated that problem solving helps them to develop various skills such as logical thinking, application of knowledge and cognitive development. This is important because the examples of the skills that were given by the pre-service teachers are the same that problem solving does in fact develop

(Hardin, 2002; Dogru, 2008). Several participants did not respond to the question about whether or not they enjoyed problem solving. This could indicate that either they simply missed the question or did not feel that problem solving helped them and therefore consciously did not answer the question. Eight percent stated that engaging with problem solving "makes" (indicating it compels) one to explore different ways of solving a problem. This made it seem as though they enjoyed the challenge that problem solving presented. This suggests that if pre-service teachers enjoy solving problems that they may present it in a positive way to their future learners.

- **Pre-service teachers' understanding of problem solving**

Three pre-service teachers indicated that problem solving to them means using some type of 'scientific method to find the cause of the problem'. This is an unusual understanding of problem solving because Science and Mathematics learners are not expected to find the 'cause' of problems. The aim of problem solving is explained by Mourtos, Okamoto and Rhee (2004) "as a process, used to obtain a best answer to an unknown, or a decision subject to some constraints" (p.1). Another definition of problem solving is offered by Dogru (2008), who states that "[s]olving a problem means to find or create new solutions for the problem" (p.9). Neither in these two definitions nor others that have been encountered in this study has there been mention of 'finding the cause of problem' as an expectation of problem solving. Five pre-service teachers said that problem solving to them meant finding an answer to a specific problem. Additionally, they said that problems may vary. Four pre-service teachers stated that problem solving requires using the capabilities one has, reflecting on cognitive development, and using critical thinking. These skills that the pre-service teachers felt were developed by using problem solving are the skills that problem solving does aim to develop.

The participating pre-service teachers responded with a single word, either 'yes', or 'no' to the question about whether they had sufficient skills to solve problems. These options were not given, but all the pre-service teachers chose to respond similarly. Two out of twelve pre-service teachers indicated that they did not feel that they had sufficient skills to solve problems. Five out of twelve pre-service teachers indicated that they did feel they had sufficient skills to solve problems. This does raise a concern because it was expected that all twelve pre-service teachers should be prepared with adequate skills, not only to engage with problem solving but also to teach these skills.

Inadvertently, we would need to question the teacher training program if these pre-service teachers cannot show sufficient confidence in their problem solving skills. It is more than likely that insufficient time is being spent on formal training in problem solving. This was expected because the pre-service teachers in the study were all in their third or final year of study and it was expected that by that stage in their training they would be confident in their skills as problem solvers. The two who indicated that they did not feel they had sufficient problem solving skills stated, after much probing during interview one, that they could do more to improve their problem solving skills, hence their response to the question in the questionnaire.

- **Pre-service teachers' perceptions of their skills to teach problem solving**

Four pre-service teachers stated that they thought they had sufficient skills to teach problem solving. Three pre-service teachers stated that they did not think they had sufficient skills to teach problem solving and five did not answer the question. It is noteworthy that 41% did not attempt the question. This suggests that either the pre-service teachers did not understand the question, forgot to answer the question, or simply did not want to answer it, or did not want to declare their lack of skills in solving problems. Thirty three percent of the participants indicated that they did have sufficient skills to teach problem solving and one pre-service teacher said that he had sufficient skills to solve problems up to high school level. Twenty five percent indicated that they did not think they had sufficient skills and one pre-service teacher added that she would like to develop these skills further. This may be an important request that ought to be engaged with further. Perhaps a module dedicated to the teaching of problem solving should be established. It is difficult to ascertain whether problem solving skills had been mastered or not. Nonetheless, it should be of concern to teacher training institutes that pre-service teachers may not possess the skills to solve problems.

- **Pre-service teachers' views on the importance of teaching problem solving**

Eleven pre-service teachers said they thought it was important to teach problem solving. This is significant because it means that 91% of the participating pre-service teachers recognised the benefits of teaching problem solving and therefore deemed it important to teach problem solving to their learners. Perhaps they used the skills widely in their academic lives hence they understood its importance. One pre-service teacher indicated that he did not think it was important to teach problem solving. It was explained that a skill is something you cannot teach but it is developed through practice. In essence, the pre-service teacher agreed that problem solving should be taught but can only be attained through practice. This is indeed debatable because there are various modules at universities and presentations at schools that focus on problem solving. Problem solving is mastered through practice but the skill can be initially taught.

- **Theme 2: Pre-service Science and Mathematics teachers' understanding of visualisation.**

This theme arose from responses to the questionnaire. A discussion on the responses of each related question follows:

- **Working with visualisation strategies to solve problems**

Eleven pre-service teachers stated that they had used visualisation strategies to solve a problem. One out of the twelve pre-service teachers said that he was not sure if he had used visualisation strategies to solve a problem. Ninety two percent of the participating pre-service teachers used visualisation strategies to solve problems, indicating that they were aware of what visualisation strategies are. Some of the pre-service teachers added that the visualisation strategies that they had used were ones that they were exposed to in their university studies. All the pre-service teachers involved in this study were enrolled in Science and Mathematics modules at university, which incorporated visualisation strategies and problem solving in their curriculums. Therefore, ideally, all the pre-service teachers should have responded 'yes' to using visualisation strategies to solve a problem. There was one pre-service teacher who indicated that he did not use visualisation strategies to solve problems. However, the pre-service teacher added that he was not sure if PowerPoint was a visualisation strategy.

- **Attitudes towards using visualisation strategies**

All the pre-service teachers said that they enjoyed using visualisation strategies. The most common reason that eleven out of twelve pre-service teachers gave for enjoying the use of visualisation strategies was that it made the problem they were working with easier to solve. In retrospect, the researcher should have probed further about why and how the use of visualisation strategies made it easier to solve problems. One of the pre-service teachers gave no reason for enjoying visualisation strategies. This means that the pre-service teachers saw the advantages of using visualisation strategies in relation to problem solving. This result could mean that if they could see the value in visualisation strategies to solve problem, then perhaps they would carry the use of visualisation strategies forward in their future classrooms.

Pre-service teachers' attitudes towards using visualisation strategies may be linked to whether or not they find these strategies useful. All the participating pre-service teachers specified that visualisation was useful because it helped them solve problems. Their reasoning differed, however, the general consensus was that visualisation helped them to make sense of the data given. 'Seeing' the data in a picture seemed to help the pre-service teachers to understand the problem. This enabled them to plan the solution and saved time in solving problems. Pre-service teacher 2 made a comment that "*using visualisation helps me get insight into the nature of the problem*". Pre-service teacher 2, in essence, is stating that using visualisation strategies gave her a deeper understanding of problems. It is hoped that if pre-service teachers can determine (by introspection) how using visualisation strategies helps them, then they will teach these strategies to their learners. Visualisation strategies are the methods/tools used to solve problems. "A visualisation method is a systematic, rule-based, external, permanent, and graphic representation that depicts information in a way that is conducive to acquiring insights, developing an elaborate understanding, or communicating experiences" (Lengler & Eppler, 2006, p.1).

- **Pre-service teachers' understanding of visualisation**

'Making a mental picture' is how six pre-service teachers explained what visualisation meant to them. This is a crucial comment because in essence, 'making a mental picture' is part of what visualisation entails. Therefore, 50% of the pre-service teachers moderately understood what it means to visualise. One pre-service teacher said that visualisation is a way of learning by predicting what a picture would be by looking at the data. Another pre-service teacher said that visualisation means to analyse data given. Alternatively, one pre-service teacher said that to her, visualisation meant to add a picture to a problem hence making the problem easier to understand. This is a fundamental point as the pre-service teacher relayed that she had difficulty in understanding the problem in written word, however, the use of visualisation strategies helped her to make sense of the problem. It is significant to note that this pre-service teacher spoke English as her mother tongue. Therefore, understanding the language should not have been a problem, but what this case highlights is the difficulty of the language, or jargon, of Science and Mathematics. The question that this leads to is, if a first language English speaker has difficulty comprehending a problem, how much more difficult is it for students whose first language is not English? The pre-service teacher indicated that using a diagram helped her understand the problem better. This is perhaps a suggestion that can be used by English second language speakers when solving problems.

Another pre-service teacher said that visualisation to him meant getting the details of a question and making sense of the question. Similarly, another pre-service teacher said that visualisation means to solve a problem practically by developing a visual picture of the problem and having a physical representation. The 'physical representation' mentioned by this pre-service teacher could refer to objects/mediating artifacts. Arcavi (2003) defines visualisation as "the ability, process and the product of creation, interpretation, use of and reflection upon pictures, images and diagrams in our mind" (p.217). In linking the pre-service teachers' understanding of visualisation and what an expert in the field defines visualisation as, it can be concluded that these pre-service teachers had a fair understanding of the concept of visualisation within a teaching context.

- **Pre-service teachers' views about the importance of teaching visualisation strategies**

All the pre-service teachers stated that they thought it was important to teach visualisation strategies, however, their reasoning for this differed. The majority of the pre-service teachers indicated that they would teach their future students about visualisation strategies because it would make it easier for students to understand problems.

Pre-service teacher 2 reasoned that it is important to teach problem solving because it "*allows them [learners] to develop abstract thinking without concrete models*". The 'concrete models' that pre-service teacher 2 mentioned are the same as mediating artifacts. Not all institutes would have mediating artifacts to assist learners to understand a problem and, in these cases, learners would have to imagine certain aspects of a problem in order to understand it better. Her comment perhaps would have had more meaning when looked at it in the context of an underprivileged school. Pre-service teacher 3 explained that he considered visualisation strategies important to be taught as "*being able to use different approaches to teaching can uplift mind-setting of learners*". By 'mind-setting' one can assume that the pre-service teacher meant to refer to the thinking of learners as the cognitive development of learners.

- **Theme 3: Pre-service Science and Mathematics teachers' understanding of the link between problem solving and visualisation.**

This theme arose from responses to the following questions from the questionnaire. A discussion of the responses of each related question follows:

- **Pre-service teachers' views about visualisation strategies that can be used to solve problems.**

The participating pre-service teachers were asked to list any type of visualisation strategies that in their opinion can be used to solve problems. In retrospect, this question could have been phrased more clearly, asking for more details. Perhaps it should have been worded as, "List visualisation strategies that one could use to solve problems at a specific level and topic of your choice". In this way, the responses could have been narrowed down and perhaps the responses would have been more meaningful. Lenger and Eppler (2007) categorised 100 visualisation strategies into six groups, namely, Data Visualisation, Information Visualisation, Concept Visualisation, Strategy Visualisation, Metaphor Visualisation, and Compound Visualisation. Most of the examples that the pre-service teachers gave can fit into one of the groups supplied by Lenger and Eppler. However, some examples given were not visualisation strategies, but rather examples of skills. Such examples of skills were as follows: visual skills, decision making skills, practical skills, critical thinking, listening skills, observing, measuring, analysing, predicting, interpreting, and questioning. The examples that were fitting to visualisation strategies were demonstrations and drawings.

All the pre-service teachers indicated that they thought there was a link between problem solving and visualisation strategies, although their reasoning differed. The majority of the pre-service teachers specified that the link between problem solving and visualisation strategies was a dependable one. They further explained this by stating that solving a problem quicker and efficiently depended on the type of visualisation strategy or strategies used. Overall, the use of visualisation strategies helps to better understand and solve a problem. On the one hand, pre-service teacher 1 described the link between problem solving and visualisation in terms of motivation. He explained that using visualisation strategies to solve a problem made it easier to solve, hence motivating one to solve more problems. On the other hand Pre-service teacher 5 gave a more practical explanation of the link between the two. She articulated that the link between problem solving and visualisation is that both working together aim to search for the solution to a problem.

What emerges is that pre-service teachers closely observed their cognition. Their consciousness of visual stategies employed to think deeply about problem was raised.

4.2.2 Research question 2

Research question two was: *Why do pre-service Science and Mathematics teachers who use visualisation strategies choose these strategies when teaching problem solving?* Two themes materialised from this research question, they are:

- Visualisation strategies that pre-service Science and Mathematics teachers use when solving problems.
- Reasons why these pre-service Science and Mathematics teachers use the visualisation strategies that they use.

These themes emerged from two task sheets that were given to the participating pre-service teachers as a data generation tool. The Mathematics task sheet had four problems to be solved and the Science task sheet had three. In retrospect, there should have been an equal number of problems for both task sheets. The questions were selected purposively on the basis that the questions lent themselves to the possibility of using various strategies to solve them. Furthermore, the questions were chosen in accordance with what knowledge the pre-service teachers should have had in order to teach problem solving to their learners. The curricula of two modules (Natural Science Method 2 and Mathematics Method 2) that all the pre-service teachers were enrolled for (among other modules) incorporated problem solving. The level of difficulty that the pre-service teachers were trained for matched that of the questions selected for the task sheets. Each of the questions comprised two sub-questions. The sub-questions required them to first solve the problem and then to explain their choice of strategy used. All the responses were analysed. The focus of the analysis was not to check on the ability of pre-service teachers to solve problems, but rather to gain insight about the strategies used and their ability to reflect on it. The reason for this was based on the hypothesis that if pre-service teachers can explain their choice of strategy, then in essence, they should be able to explain it to their learners.

To recap, there were twelve pre-service teachers who completed the questionnaire. However, thereafter nine stayed on to complete the task sheets. The assumption here is that the visualisation or other strategies which pre-service teachers use in order to solve problems themselves, will be similar to those that they will teach learners to use. The pre-service teachers' responses to the Mathematics task sheet and Science task sheet will be analysed respectively. Theme 1: Visualisation strategies that pre-service Science and Mathematics teachers use when solving problems, and theme 2: Reasons why these pre-service Science and Mathematics teachers use the visualisation strategies that they use, will be discussed simultaneously working from the Mathematics task sheet to the Science task sheet.

4.2.2.1 Mathematics Task Sheet

- **Problem one as it appeared in the Task sheet:**

Nandi wants to tie a ribbon around the sides of a hexagonal box and then tie a bow. Each side of the box is 4 cm long. She needs 14 cm for the bow. Is half a meter of ribbon enough? Explain. Then state exactly how long the ribbon must be.

- **Strategies used to solve the problem**

There was only one pre-service teacher who used a combination of a diagram and formula and solved problem one correctly. Two pre-service teachers used a combination of a diagram and an equation and solved the problem correctly. Three pre-service teachers used a combination of a diagram, conversions, and an equation and solved it correctly. It is necessary to distinguish between the terms formula and equation. A formula, in the case of the Mathematics task sheet, refers to the use of an established rule that has been tried and tested and that appears in the form of words. 'Equation' in the Mathematics task sheet refers to a mathematical statement, not one that is in the form of an established rule. It might be an equation that the student derived on his or her own. Collins (2009) explains 'conversions' as "a change or adaptation in form; a change in the units or form of a number or expression" (p.373).

One pre-service teacher used the combination of a formula and reasoning and worked out the problem correctly. Collins (2009) explains reasoning to mean, "the faculty of rational argument, deduction" (p.1370). Comparably, one pre-service teacher used the same combination of a formula and conversions, however, but solved it incorrectly. It seems as though different pre-service teachers prefer a variety of combinations and it works for some. It is important to take from this that there are many variations of combinations of problem solving strategies that can be chosen from. As pre-service teachers of Science and Mathematics, it is important for them to be able to use a mixture of methods and be able to teach this technique to their learners. Most of the pre-service teachers used a diagram as well as some form of equation and arrived at the correct solution. This may indicate that these pre-service teachers had a tendency to use a combination of diagrams/visual aids and equations when answering the type of question depicted in question one. Only one out of the nine pre-service teachers worked out the question incorrectly and used a combination of a formula and conversions to do so. A significant finding emerged in the case of this pre-service teacher. In all the Mathematics problems that he solved in the Mathematics task sheet, this is the only one he had worked out incorrectly. In all the other (four out of five) problems he had used a combination of a diagram and a formula. This combination seemed to work for him. It is possible that he would have answered the question correctly if he used the combination of a diagram and a formula. In hindsight, this should have been probed further during the interview. Two pre-service teachers used a combination of a formula and either reasoning or conversions.

It is important to note that all of the pre-service teachers used multiple strategies to solve the given problems. This could suggest that neither the strategy of a diagram nor a formula proved to be adequate on its own. More significantly, though, it indicates that multiple strategies are essential in classroom teaching. With the combination of many learners in each class it is probable that these are learners who prefer different strategies in each class. It is therefore recommended that teachers, especially pre-service teachers, become au fait with multiple strategies of solving problems. Multiple strategies will cater to learners that use different strategies as a result of their multiple intelligences.

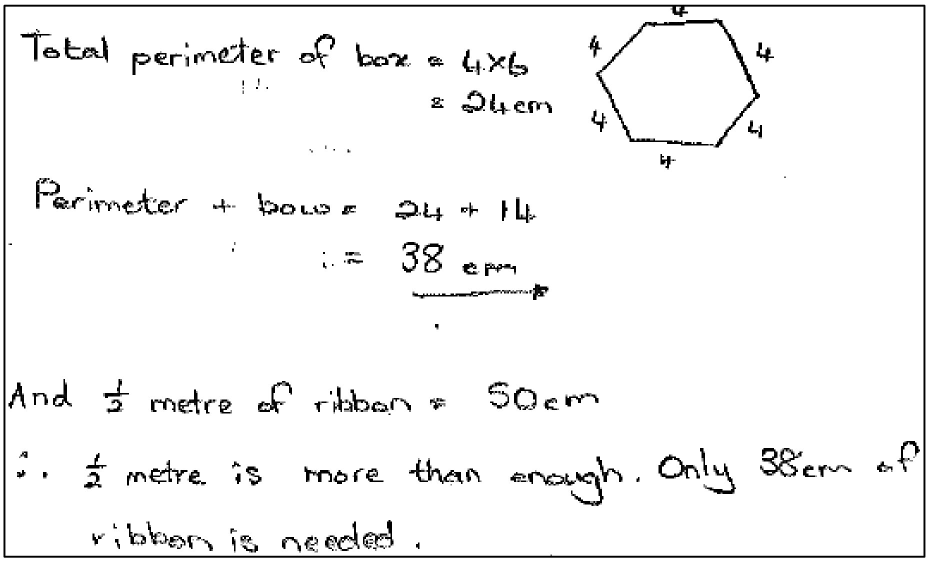


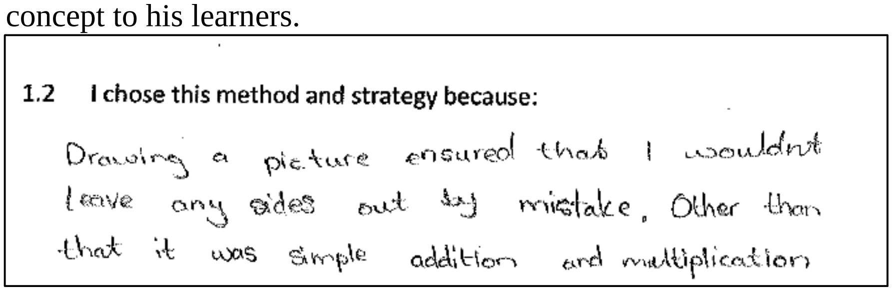
Figure 6: Mathematics Pre-service teacher 6's response to problem one of the Mathematics task sheet

Pre-service teacher 6 used a diagram as well as a formula and equation (represented by Figure 6). She solved the problem correctly. I assume that she began with the drawing of the hexagon and then proceeded to fill in the measurements from the information given in the question. This assumption is based on the fact that the diagram is placed prominently above the pre-service teacher's written work. It seems as though the diagram helped her to make sense of the information in the question. She then used the diagram to complete the formula to calculate the perimeter of a hexagon, followed by an equation that involved subtraction. These three steps brought her to the final correct answer. It is possible that this pre-service teacher drew the diagram to do two things. One was the ability to see part of the solution. By simply looking at the diagram it is possible to calculate the perimeter of the base, which was 24 cm. However, the pre-service teacher still proceeded to use a formula. This could indicate that a rigid calculation serves to convince the problem solver that the answer is correct. Second, and perhaps more important, the diagram serves to place the problem in context. Inherent in the wording of the problem are terms that needs clarity such as 'hexagonal'. If this is not known then the possibility of solving the problem is reduced.

- **Reasons for using a combination of formulae and diagrams to answer question one of the Mathematics task sheet.**

There were two main reasons that the pre-service teachers provided for using a combination of formulae and diagrams. Firstly, the pre-service teachers stated that they found it easier to get to an outcome because the diagrams simplified the question and the use of formulae helped attain an answer. Secondly, the pre-service teachers explained that it was a matter of following logical steps to arrive at the answer.

Four pre-service teachers stated that they used logical reasoning to answer the problem. This reveals that perhaps they were familiar with this strategy when solving this type of problem. All the problems chosen for the Mathematics and Science task sheets were based on the expected *a priori* knowledge of the pre-service teachers. This is based on the fact that the pre-service teachers were enrolled for modules (Science and Mathematics method 2) which included such problems. In addition to this, many of the problems they worked with could have been solved by learners at schools. The pre-service teacher eight whose reasoning I could not comprehend used a combination of a diagram, conversion, and equation and solved it correctly. This is intriguing because this pre-service teacher was soon to be a teacher and would have learners of his own and it can be argued that if he cannot explain why he used the strategies he used, he would not be able to explain such a concept to his learners.


1.2 I chose this method and strategy because:

Drawing a picture ensured that I wouldnt leave any sides out by mistake. Other than that it was simple addition and multiplication

Figure 7: Pre-service teachers six’s response to the reason for the strategy used

Pre-service teacher 6 explained that she drew a diagram to help make sense of the given information. This is the purpose of a visual aid. Her explanation may make it seem that the diagram helped to consolidate the question and the answer. It helped to build confidence because, as she claimed, she was afraid that she could have left out one or more sides. Therefore, the diagram played a vital role in the solving process.

- **Problem two as it appeared in the task sheet.**

The length of a rope, to which a cow is tied, is increased from 19 m to 30 m. How much additional ground will it be able to graze in? Assume that the cow is able to move on all sides with equal ease.

- **Strategies used to solve problem two of the Mathematics task sheet.**

Seven out of the nine pre-service teachers used a combination of a diagram and an equation and solved it correctly. The same combination of a diagram and equation was used by two out of the nine pre-service teachers, but they had solved it incorrectly. It is important to note that all the pre-service teachers used a diagram to solve the problem. It seems that their initial response to the solution of any problem is the use of some diagram. Pre-service teacher 10 and Pre-service teacher 12 provided incorrect solutions for the problem. Pre-service teacher 10 had drawn a diagram with the given values in place, however, he failed to differentiate between the inner and outer circles of where the cow could graze. This is important because it might indicate that he had not conceptually understood the idea of areas in and between concentric circles. He only calculated one of the grazing circles. It is possible that had he drawn a clearer diagram, he may have noticed the two grazing paths and this would have presented him with a deeper understanding. Pre-service teacher 12 had an accurate diagram with measurements, however, his calculations were incorrect.

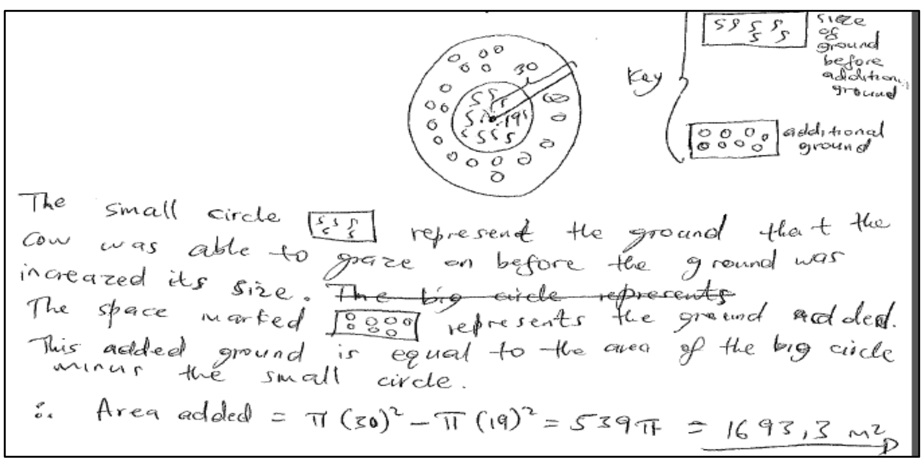


Figure 8: Pre-service teacher 8's strategy to solving problem two of the mathematics task sheet

Pre-service teacher 8 used a visual aid as well as an equation to solve the problem accurately (represented by Figure 8). The visual aid comprised various aspects such as two concentric circles which have symbols indicating the two paths for the cow to walk in, a key explaining the two symbols used by the Pre-service teacher, the single radius which indicates that he recognised that there was an increase in the radius and the use of the parenthesis further indicates the increase of the radius. The diagram that Pre-service teacher 8 provided was fairly self-explanatory and would have been sufficient had he not provided the additional explanation using words. The essence of what he was thinking was sufficiently captured in his diagram and may have, if used with learners at school, provided a useful starting point for their own solutions. This is perhaps one of the most significant uses of diagrams when solving problems (not necessarily in Mathematics only). Furthermore, in recognising that the circles had a common centre (as indicated by his diagram). Pre-service teacher 8 produced what may be termed as a 'visual solution' (Lowrie & Kay, 2001). By simply looking at the diagram, it can be observed where the solution was located and to some extent, how one can find the solution.

- **Reasoning for strategies used to answer question two of the Mathematics task sheet.**

All of the pre-service teachers provided reasons for the strategies that they used. One was unclear and was therefore omitted. Six out of nine pre-service teachers indicated that they used a diagram because it made understanding the measurements and hence solving the problem easier. This, in essence, implies that through the construction of the diagram, they were able to better conceptualise the problem and this may indicate an increased ability to solve the problem. Two of the pre-service teachers indicated that they used diagrams due to their need to create logical reasoning. Despite this claim, their reasons for the use of this strategy were very similar to the other six pre-service teachers because any form of mathematical argument should be construed as logical reasoning. Nonetheless, all the pre-service teachers used diagrams as a tool to assist in solving the problem. The important point that ought to be emphasised is that six of the pre-service teachers indicated that the diagram enabled them to 'see' the solution. This is perhaps significant because it shows the strength of the visual, as compared to only using the algebraic method.

The one pre-service teacher who also provided the correct solution could not explain why he used this particular method of drawing a diagram. This inability to provide a coherent reason is of concern because he may actually struggle when teaching his own learners.

- **Problem three as it appeared in the task sheet.**

Find the equation of the tangent at (0,2) to the circle with equation: $(x + 2)^2 + (y + 1)^2 = 13$

- **Strategies used to solve problem three of the Mathematics task sheet.**

This question was also answered correctly by all the participants. Seven out of nine pre-service teachers used both diagrams and formulae, whilst the remaining two only used formulae. Perhaps it is important to indicate that this question could have easily been solved using just a formula, but the majority of participants chose to use diagrams as a strategy. Again, their reason for choosing to work with diagrams was that visualisation made obtaining the solution easier. This emphasises the idea that ‘seeing’ the solution is as important as obtaining the solution. One of the four pre-service teachers indicated that they used ‘logical reasoning’. It can be argued that all the pre-service teachers in fact used logical reasoning as a strategy, including those who used diagrams, because logical reason is about presenting a logical argument. Logical arguments can easily be presented visually as well and therefore it is argued that all pre-service teachers actually used logical reason.

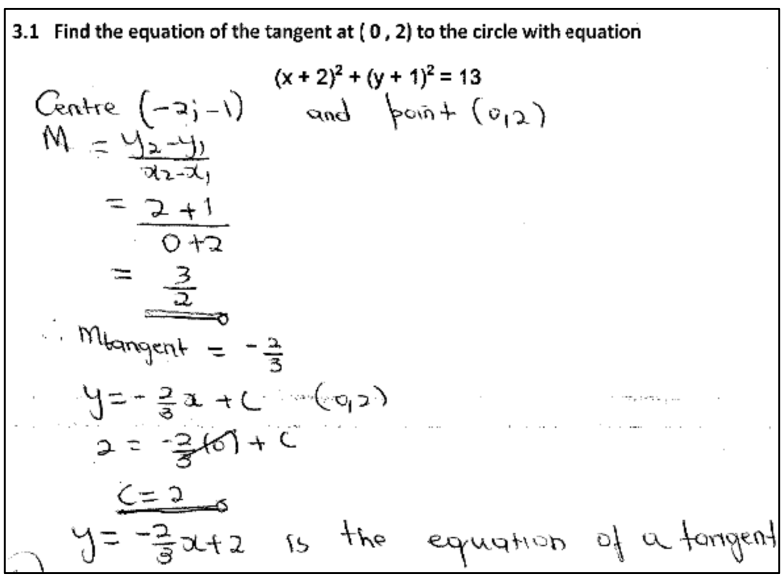
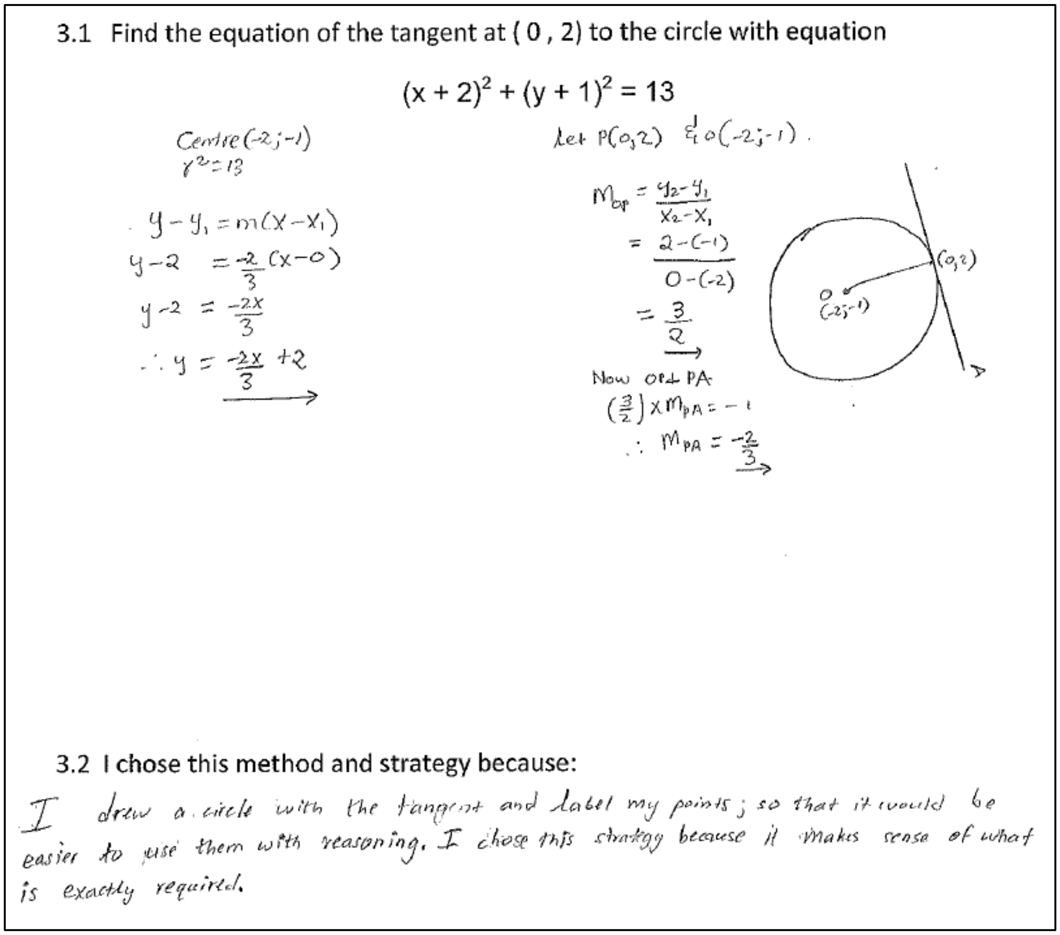


Figure 9: Pre-service teacher 10’s response to problem three of the Mathematics task sheet

Figure 9 presents Pre-service teacher 10’s response to problem three of the Mathematics task sheet. Pre-service teacher 10 was one out of the two pre-service teachers who did not draw a diagram, but solved the problem accurately. It is possible that he was familiar and confident with such a problem, hence he probably did not need the support or clarification that the use of a diagram would provide. This may also indicate that he was fairly familiar with this type of problem and could have easily created a mental picture. I contend that the lack of evidence of understanding through drawn diagrams does not mean that visualisation strategies were not involved; indeed, the participant could have had a mental image of the problem.


3.1 Find the equation of the tangent at (0 , 2) to the circle with equation

$$(x + 2)^2 + (y + 1)^2 = 13$$

Centre (-2; -1)
$r^2 = 13$

$y - y_1 = m(x - x_1)$
$y - 2 = \frac{-2}{3}(x - 0)$
$y - 2 = \frac{-2x}{3}$
$\therefore y = \frac{-2x}{3} + 2$

Let P(0; 2) & O(-2; -1).

$M_{OP} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{2 - (-1)}{0 - (-2)}$
$= \frac{3}{2}$

Now OP ⊥ PA
$(\frac{3}{2}) \times M_{PA} = -1$
$\therefore M_{PA} = \frac{-2}{3}$

3.2 I chose this method and strategy because:

I drew a circle with the tangent and label my points; so that it would be easier to use them with reasoning. I chose this strategy because it makes sense of what is exactly required.

Figure 10: Pre-service teacher 3's working out of problem three of the mathematics task sheet followed by a rational

Pre-service teacher 3 used a strategy (in Figure 10) which was different to that of pre-service teacher 10 (Figure 9) because Pre-service teacher 3 drew a diagram. However, it is important to note that both pre-service teachers solved the problem correctly. As the pre-service teacher explained, he used a diagram in conjunction with a formula because it made sense of the data given.

- **Reasoning for strategies used to answer question three of the Mathematics task sheet.**

Five pre-service teachers stated that they used diagrams because it gave the information provided greater meaning when they could see it on a drawing. Additionally, four pre-service teachers stated that they used logical reasoning to answer the problem and that it was a problem that they were familiar with. It was evident that they were familiar with the problem because two of these four pre-service teachers used only a formula and solved the problem correctly. It can be assumed that the more complex a problem is, the more likely pre-service teachers are to use a diagram to help make sense of the data given and then to solve the problem.

- **Problem four as it appeared in the task sheet**

The triangle bounded by the lines y = 0, y = 2x and y = -0.5x + k, with k being positive, is equal to 80 square units. Find k.

- **Strategies used to solve problem four of the Mathematics task sheet**

In solving this problem, the pre-service teachers used two methods. First was the use of a diagram and formula. Seven pre-service teachers used this combination and they were the only ones who solved the problem correctly. The same was found for questions two and three of the Mathematics task sheet. The same combination (diagram and formula) was used by Pre-service teacher 6 who solved the problem incorrectly. The second method, which was the use of an equation, was used by just one pre-service teacher, Pre-service teacher 4. It was clear that she misunderstood the question because she solved for the area of a triangle instead for the value of 'k'. This was significant because, like all the other questions that the pre-service teachers had to answer, this too was one that they should have been familiar with. However this was the only question that pre-service teacher 3 solved incorrectly in the Mathematics task sheet.

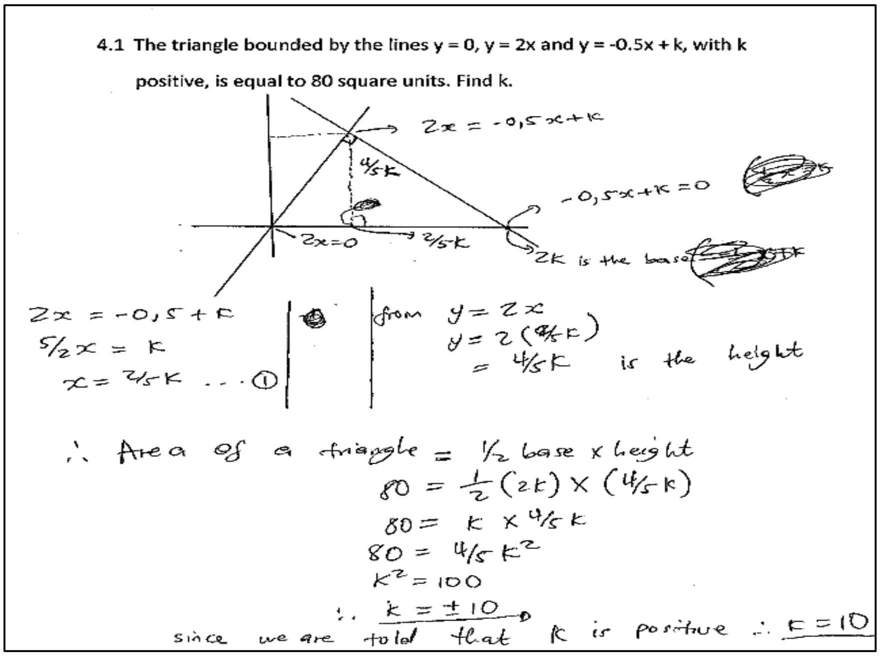


Figure 11: Pre-service teacher 8's response to problem four of the Mathematics task sheet

Pre-service teacher 8 adopted a solution strategy which was similar to that used by most of the other pre-service teachers. His scribbling to the right of the diagram shows a deeper interaction with his visual aid. Perhaps we see a further use of the visual aid. Besides providing a summary of all the information provided, it also offers the user an opportunity to add more information when attempting to solve the problem. Using the diagram as a visual tool is more pronounced here due to the inclusion of the perpendicular line and the use of arrows. In a sense, the diagram assists the reader to see how the pre-service teacher thought about the solution. The lines to the axes from the intersection of $y = 2x$ and $y = -0.5x + k$ almost certainly shows the height of the triangle, this is indicated by the right angle sign on the x axis. This diagram also served as a point of reference for the pre-service teacher. As the pre-service teacher completed the calculation, he referred to the diagram and inserted information like the 'x' intercept of the line $y = -0.5x + k$. This type of solution demonstrates the value of diagrams. Its value should be gauged from the relevant interaction between the solver and the diagram. This pre-service teacher's interactive use of the diagram during the solution may have also assisted in understanding the problem.

- **Reasoning for strategies used to answer question four of the Mathematics task sheet.**

Six pre-service teachers stated that they used a diagram as it made the problem easier to solve. It is significant that although only six pre-service teachers stated that they used a diagram because it made it easier for them to answer the question, eight out of the nine pre-service teachers used a diagram when finding the solution to the problem. Two pre-service teachers explained that they used logical reasoning to arrive at the answer. The remaining two pre-service teachers explained that they used logical reasoning as the question was one that was familiar to them. Only one out of the nine pre-service teachers did not complete the problem or the reason section of the worksheet. The pre-service teacher who did not finish explained that she tried her best and thought that the diagram would have helped, but it did not, therefore she did not complete the section. What is interesting is that this pre-service teacher (Pre-service teacher 6) still attempted the question and the diagram was her last effort. This pre-service teacher perhaps showed that just having a diagram is insufficient. There needs to be sufficient interaction between the solver and the diagram. The diagram, if drawn correctly, may help with the conceptualisation of the problem in order to find a correct solution.

Table 7: Summary of strategy or strategies used to answer the Mathematics task sheet

Pre-service teacher	Question 1					Question 2					Question 3					Question 4				
	Diagram	Equation	Formula	Correct?	Reason. Made it easier to explain?	Diagram	Equation	Formula	Correct?	Reason. Made it easier to explain?	Diagram	Equation	Formula	Correct?	Reason. Made it easier to explain?	Diagram	Equation	Formula	Correct?	Reason. Made it easier to explain?
2	✓		✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓
3			✓		✓	✓		✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓
4	✓	✓		✓	✓	✓		✓	✓	✓	✓		✓	✓	✓		✓			✓
5		✓	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓		✓		✓	✓	
6	✓	✓		✓	✓	✓		✓	✓		✓		✓	✓	✓	✓		✓	✓	
7	✓	✓		✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓		✓		✓
8	✓	✓		✓		✓		✓	✓	✓			✓	✓	✓	✓		✓	✓	✓
10			✓	✓		✓		✓					✓	✓		✓		✓	✓	
12	✓	✓		✓	✓	✓		✓		✓	✓		✓	✓	✓	✓		✓	✓	✓

Table 7 depicts the nine pre-service teachers and the four questions they answered, along with what strategy they used and if they solved it correctly. The table also shows the reasons they chose the strategies that they chose. It should be noted that the pre-service teachers were numbered one to twelve, but three pre-service teachers (Pre-service teacher 1, Pre-service teacher 9, and Pre-service teacher 11) had left the study at this point and did not complete the Mathematics task sheets, hence the numbering does not correlate. The sign "✓" indicates a method used and if they got the question correct. Empty cells indicate methods not used and if they got the question incorrect.

The finding which emerges from Table 7 is that the majority of the participating pre-service teachers who used both diagrams and equations to solve the problems solved it correctly as compared to the pre-service teachers who did not. Also, the majority of the pre-service teachers indicated that using a diagram made it easier for them to solve the problem. From Table 7 it is evident that there is a tendency for pre-service teachers to create diagrams when solving problems. This may indicate that pre-service teachers may be visual learners in Mathematics. Closely associated with the usage of a diagram is the pre-service teachers' gravitation towards using formulae. Not many chose to create equations or use other algebraic methods.

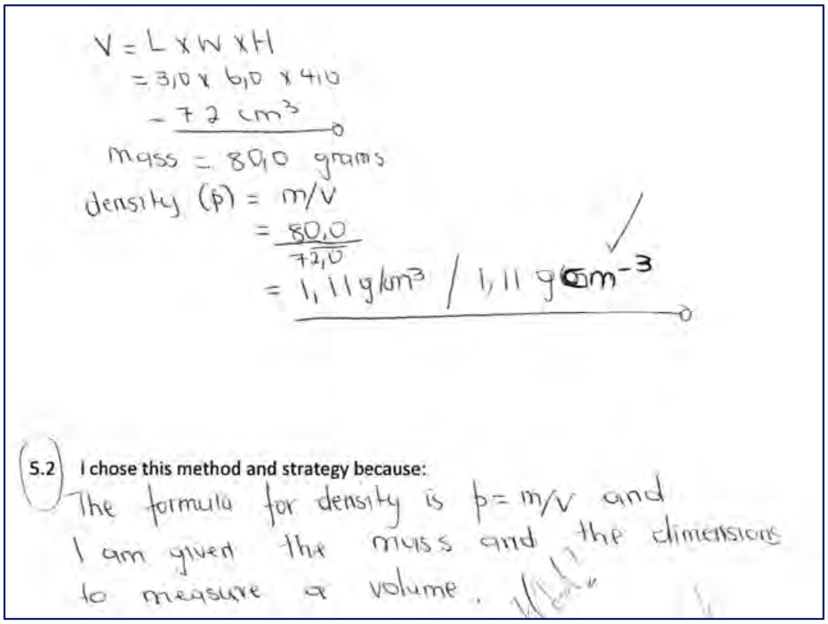
4.2.2.2 Science Task Sheet

- **Problem one as it appeared on the Science task sheet.**

A piece of wood that measures 3.0 cm by 6.0 cm by 4.0 cm has a mass of 80.0 grams. What is the density of the wood? Would the piece of wood float in water? (Volume = L x W x H)

- **Strategies used to solve the problem.**

The common strategies used to solve the problem were either a diagram and/or a formula. Three pre-service teachers used a combination of diagrams and formulae and solved the problem correctly. Two pre-service teachers used only a formula to answer the problem and calculated it correctly. However, another pre-service teacher also only used a formula but solved the problem incorrectly. The remaining three pre-service teachers did not complete the problem. Incomplete here would mean that either they did not know how to do it or they could not draw a conclusion. The common denominator in these three cases where there were "incomplete" formulae was the strategy used. It is significant to note the poor performance of these pre-service teachers on problems that should be familiar to them, especially because they were a year away from being qualified teachers.



V = L x W x H
= 3,0 x 6,0 x 4,0
= 72 cm³

mass = 80,0 grams

density (ρ) = m/V
= 80,0 / 72,0
= 1,11 g/cm³ / 1,11 gcm⁻³

5.2 I chose this method and strategy because:
The formula for density is ρ = m/V and I am given the mass and the dimensions to measure a volume.

Figure 12: Pre-service teacher 10's calculation of problem one of the science task sheet

He worked solved problem one correctly but did not complete the second part of the question (represented by Figure 12).

- **Reasons for usage of strategies to answer question one of the Science task sheet.**

The two main strategies used were diagrams and formulae. Three pre-service teachers indicated that they preferred using a diagram as it made solving a problem easier for them. They furthermore stated that diagrams made the problem easier to understand when different variables were given. Four pre-service teachers who used formulae explained that it seemed more logical than drawing a diagram. These pre-service teachers explained that they found no need for a diagram because the information given only required a formula to solve the problem. Two out of the nine pre-service teachers' reasoning was not written legibly, therefore it was grouped and labelled as unclear. The two pre-service teachers whose reasoning was unclear were second year students, and the higher education institute that they were attending used English as the language of instruction. They would have had two years of being taught in the medium of English and still they were unable to explain why they had chosen specific strategies. This could be an additional reason that explains why the Mathematics and Science results are poor in South African schools. It is possible that pre-service teachers' poor understanding of English, and their inability to explain their strategies coherently, results in ineffective teaching practices. These pre-service teachers, when qualified, are expected to teach

Science and Mathematics learners. If they are unable to express themselves while at a higher education institute, it is possible that they would be unable to do so in a school environment.

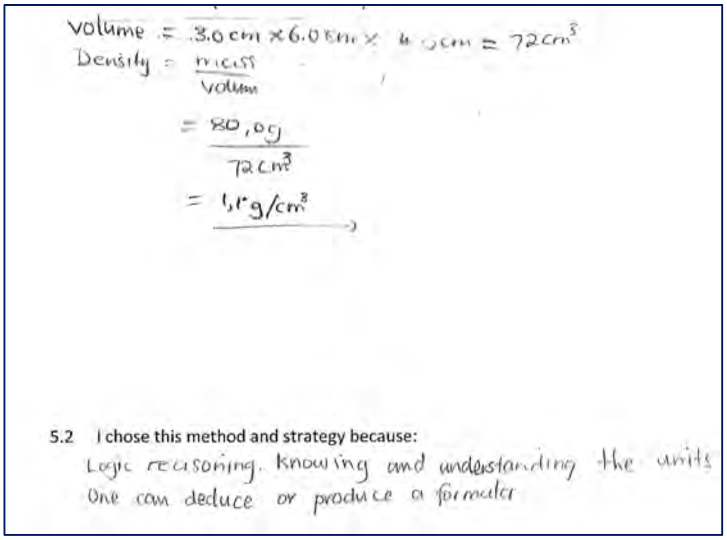


Figure 13: Pre-service teacher 12's solution and explanation of strategy used to solve problem one of the science task sheet

Pre-service teacher 12 did not complete the entire question (Figure 13) because he did not answer the final question which was, "Would the piece of wood float?" However, he explained that he used logical reasoning to solve the problem. Pre-service teacher 12 explained his understanding of 'logical reasoning' as a blend of knowing and understanding that led him to deduce an answer. In essence, 'logical' and 'reasoning' have the same meaning, which shows that the pre-service teacher did not have a clear understanding of these terms. In terms of not answering the final question of the problem, which was, "Would the piece of wood float?" it may pose a future problem for this pre-service teacher as he may not be able to ascertain if he has completed a problem. One has to wonder if he would be able to prevent his learners from making the same mistake.

- **Problem two of the Sciencetask sheet.**

A cup of gold coloured metal beads was measured to have a mass of 425 grams. By water displacement, the volume of the beads was calculated to be 48.0 cm^3. Given the following densities, identify the metal.

Gold: 19.3 g/mL   Copper: 8.86 g/mL   Bronze: 9.87 g/mL

- **Strategies used to answer question two of the Science task sheet.**

All the pre-service teachers used a formula to solve this question and they all solved it correctly. This was expected because the question lent itself to being answered more easily using formulae than by the use of a diagram. However, it was of interest to see if any of the pre-service teachers used some kind of visual aid.

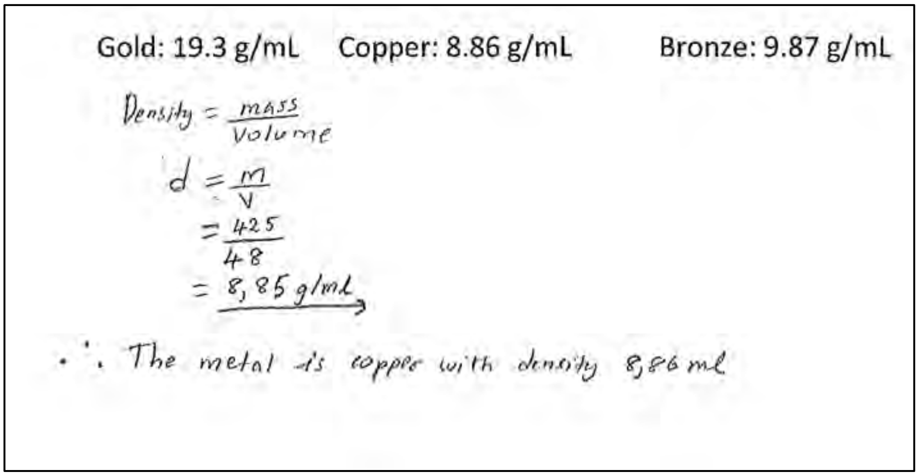
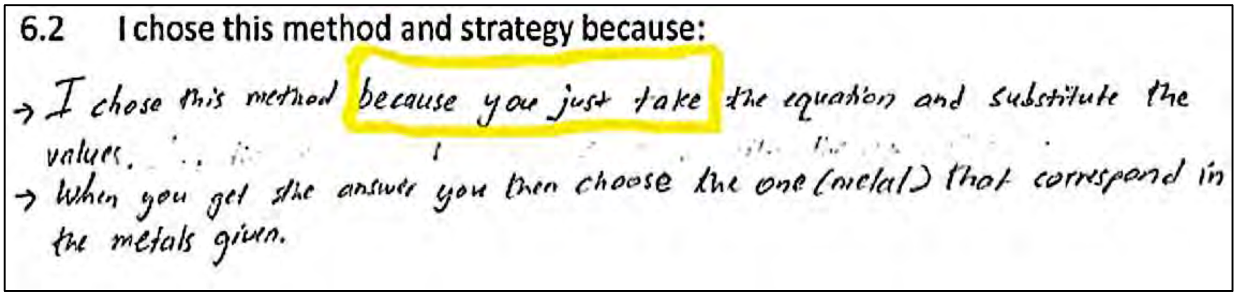


Figure 14: Pre-service teacher 3's solution to problem two of the science task sheet

Figure 14 depicts Pre-service teacher 3's solution to problem two of the Science task sheet, it is also an example of the formula that was used to solve this problem by most of the participating pre-service teachers. Pre-service teacher 3 used the density formula and substituted the appropriate values. Lastly, to determine which type of metal it was, he compared his answer to the metals given to conclude what the metal could most likely be.

- **Reasoning for the strategies used to solve question two of the Science task sheet.**

Eight out of the nine pre-service teachers stated that they used the formula to answer the problem as it seemed like the best thing to do. This was the majority of the pre-service teachers' reason for using the formula and comparison. One pre-service teacher's reason was illegible and written in poor English, hence it was labelled 'unclear'. This was the same pre-service teacher who produced unclear statements in question one of the Science task sheet.


6.2 I chose this method and strategy because:

→ I chose this method because you just take the equation and substitute the values.

→ When you get the answer you then choose the one (metal) that correspond in the metals given.

Figure 15: Pre-service teacher 3's justification for methods used to solve problem two of the Science task sheet

Figure 15 is an example of the type of response that pre-service teachers had given regarding the strategies used to solve problem two of the Science task sheet. Pre-service teacher 3's explanation (Figure 15) of why he chose to use a formula is similar to the responses from the other pre-service teachers who completed the Science task sheet. The highlighted text (in Figure 15) "because you just" indicates that the pre-service teacher used minimal effort to solve the problem due to the fact that it was so easy. It is evident that diagrams play a substantially small role when solving problems this simple. Often, a formula is recognised and the solution is obtained.

- **Problem three of the Science task sheet**

A little aluminium boat (mass of 14.50 g) has a volume of 450.00 cm^3. The boat is placed in a small pool of water and carefully filled with pennies. If each penny has a mass of 2.50 g, how many pennies can be added to the boat before it sinks?

- **Strategies used to solve problem three of the Science task sheet.**

Four pre-service teachers used a combination of formulae and reasoning to answer the problem. All four answered it correctly. One pre-service teacher used a combination of a diagram and a formula (see Figure 12). This pre-service teacher had also solved the problem correctly. Two pre-service teachers who only used a formula solved it correctly. However, two other pre-service teachers who had only used a formula to solve the problem obtained an incorrect solution.

'Reasoning' was used by four pre-service teachers and assisted them in solving the problem correctly. When answering the problem, seven pre-service teachers solved it correctly and two out of the nine solved it incorrectly. Of the seven who solved it correctly, six used a formula in addition to other strategies, and one participant used only a formula. It seems as though equations was a preferred strategy for solving this problem. Two pre-service teachers used a formula and solved it incorrectly. Perhaps a diagram would have helped this pair to solve the problem correctly. This was also the case for one of the pre-service teachers who used a formula and a diagram. It is clear that, in general, the use of formulae served the pre-service teachers well in solving the problem, yet the pre-service teachers did not find the use of formulae to be an adequate strategy to solve the problems and hence the five participants who solved it correctly used formulae in addition to diagrams or 'reasoning'. In total, only three strategies were used to solve this problem, this is a low number because all the pre-service teachers would have been exposed to many different problem solving strategies in their Science and Mathematics method modules. A possible reason is that they were familiar with these strategies or it may be that the problems were very simple and did not require additional strategies. Perhaps it should be stated that, although not part of the scope of this research, it would have been useful to investigate what strategies the pre-service teachers would use to explain to their learners when solving these problems. It might be that the strategy used to solve a problem may differ from that used to explain the solution to learners.

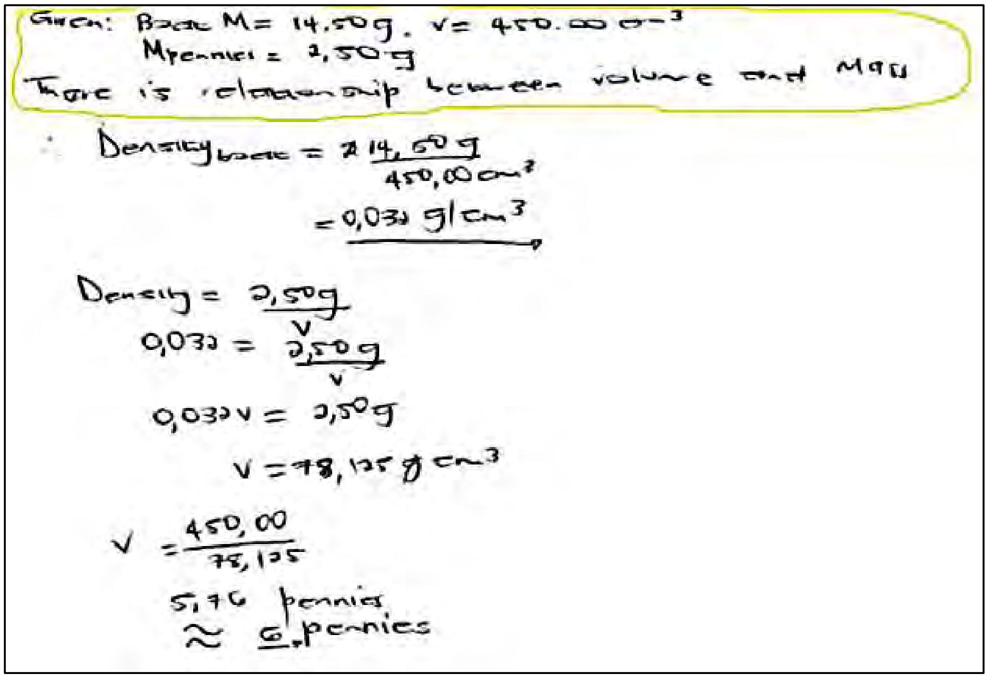


Figure 16: Pre-service teacher 4’s solution of problem three of the science task sheet

Figure 16 represents the solution to problem three by pre-service teacher 4, and shows that her use of the density formula is similar to that of the rest of the pre-service teachers in solving this problem in the Science task sheet. Significantly, in Pre-service teacher 4’s solution she showed how she had made sense of the question by indicating data that was ‘given’ and data that she could infer based on the data given. This is shown by the yellow heighted area in Figure 16. It is possible that Pre-service teacher 4 was influenced by her Mathematics based knowledge when working with this problem. The idea of indicating what is given is a useful strategy. The actual act of drawing a diagram puts the problem into perspective and it seems that it enables the problem solver to reflect on the solution as the drawing unfolds. In Pre-service teacher 6’s solution (Figure 17), the diagram does not seem to influence the solution, but the mental processes which led to the diagram being drawn cannot be determined.

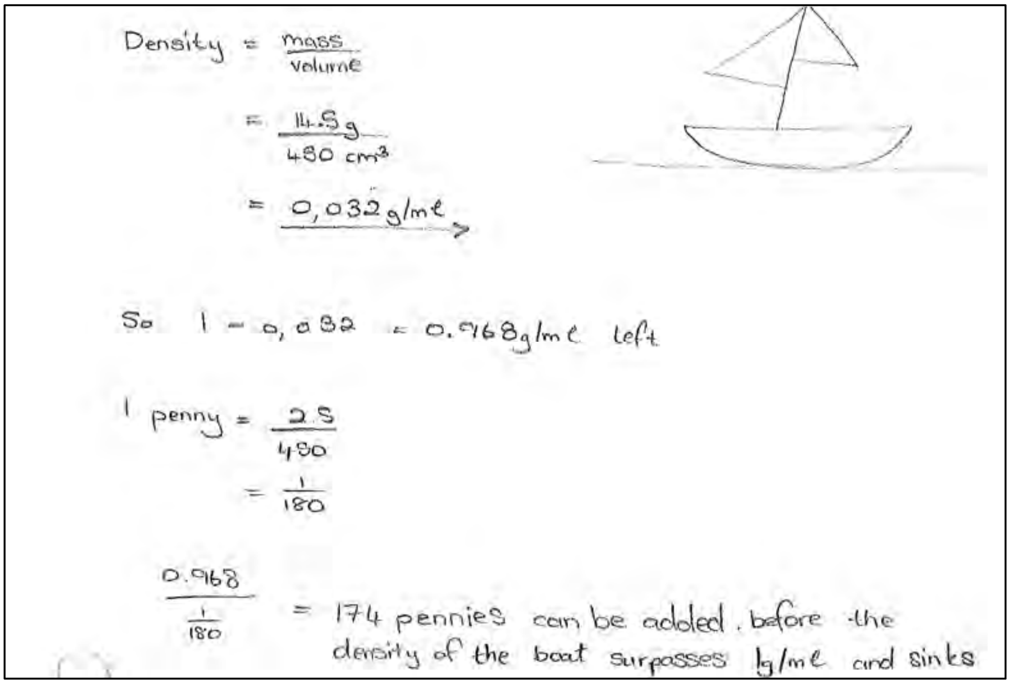


Figure 17: Pre-service teacher 6's solution of problem three of the science task sheet

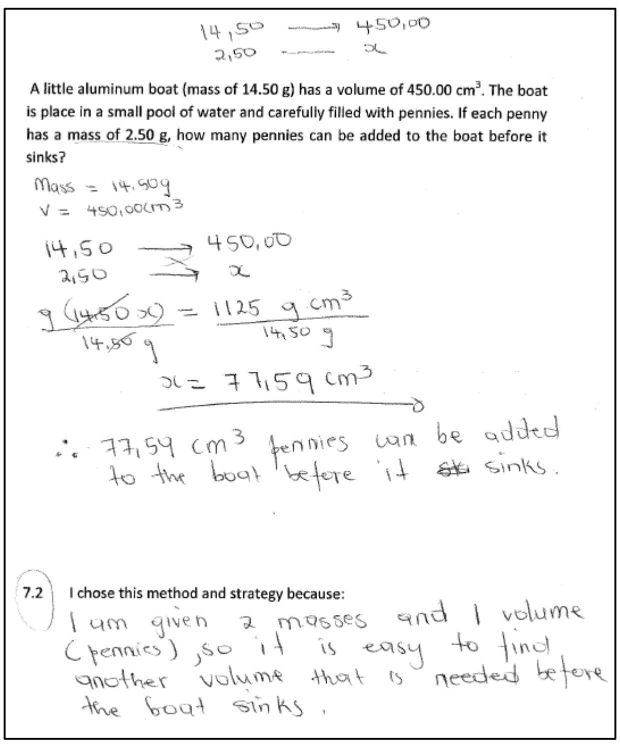
Figure 17 displays Pre-service teacher 6's solution of problem three of the Science task sheet. Pre-service teacher 6 used a combination of a diagram and a formula to solve the problem. However, the diagram seems to have been of no assistance in solving the problem. Alternatively, it is possible that the diagram may have been a means to understanding the problem. It seems that the act of drawing enabled Pre-service teacher 6 to clarify the question and provide a perspective that helped with the solution.

- **Reasoning for the strategies used for problem three of the Science task sheet.**

Five pre-service teachers explained that the reason they used a formula was because it was the logical thing to do. This ought to be translated to: this was the best option to use because the formula was immediately recognised and recalled. Perhaps the pre-service teachers' prior teaching influenced their method. They added that it was a simple case of substituting into an equation. One pre-service teacher explained the steps they had used in researching the answer. In explaining the steps used, this pre-service teacher did not answer the question but still gave meaningful information. If the pre-service teacher could explain the steps used to answer the question then one can presume that this pre-service teacher may be able to explain it in a similar way to their students.

This is just as important as being able to explain why the pre-service teacher chose the strategies that were used to answer the problem.

One pre-service teacher stated that he worked backwards using the information given. This is a new strategy that they had not used in the other problems. Working backwards is not the most mathematically correct way of solving a problem, but is no less effective than using a trial and error technique. Two pre-service teachers, Pre-service teachers 5 and 10, did not explain their reason and therefore this was labelled as 'unclear'. Pre-service teacher 5 explained that she had 'applied a procedure of floating to answer the question' this was unclear. However, her solution was correct. Again, as seen in the previous two problems of the Science task sheet, language was an issue in understanding the question, answering the question, and in explaining the answer. Pre-service teacher 10 had clearly misunderstood the question and confused mass with volume and this made his reasoning difficult to follow, as is evident in Figure 18.


14,50 → 450,00
2,50 — x

A little aluminum boat (mass of 14.50 g) has a volume of 450.00 cm³. The boat is place in a small pool of water and carefully filled with pennies. If each penny has a mass of 2.50 g, how many pennies can be added to the boat before it sinks?

Mass = 14.50g
V = 450,00cm³

14,50 → 450,00
2,50 → x

g(14,50 x) / 14,50 g = 1125 g cm³ / 14,50 g

x = 77,59 cm³

∴ 77,59 cm³ pennies can be added to the boat before it sinks.

7.2 I chose this method and strategy because:
I am given 2 masses and 1 volume (pennies), so it is easy to find another volume that is needed before the boat sinks.

Figure 18: Pre-service teacher 10's solution to problem three of the science task sheet

In Figure 18 it can be seen that it is possible that this pre-service teacher did not understand the question.

Table 8: Summary of strategy or strategies used to answer the Science task sheet

Pre-service teacher	Question 1					Question 2					Question 3				
	Diagram	Equation	Formula	Correct?	Made it easier to explain?	Diagram	Equation	Formula	Correct?	Reason. Made it easier to explain?	Diagram	Equation	Formula	Correct?	Made it easier to explain?
2	✓		✓	✓	✓			✓	✓	✓			✓	✓	✓
3			✓		✓			✓	✓	✓			✓	✓	✓
4			✓					✓	✓	✓			✓		
5			✓	✓	✓			✓	✓				✓	✓	
6	✓		✓	✓	✓			✓	✓	✓	✓		✓	✓	✓
7	✓		✓	✓	✓			✓	✓				✓	✓	✓
8			✓	✓	✓			✓	✓	✓			✓	✓	✓
10			✓	✓				✓	✓				✓		
12			✓	✓				✓	✓	✓			✓	✓	✓

Table 8 displays a breakdown of the methods that the pre-service teachers used to answer the Science task sheet. The common methods used were diagrams; formulae; and/or equations. Table 8 shows what each of the pre-service teachers used to solve each of the problems and whether they solved them correctly. If they solved a problem correctly, it is indicated by the symbol "✓". Empty cells indicate methods not used and if the pre-service teacher solved the problem incorrectly. Each pre-service teacher is numbered according to the order in which they returned their questionnaire.

The researcher's interpretation of the table is that in the Science questions that required the pre-service teachers to solve problems, the pre-service teachers preferred using formulae rather than a combination of formulae and diagrams, and they reached the correct answer by doing so. They did so because they felt that formulae were adequate and accurate strategies to use. It may be that the problems selected for the Science task sheet did not adequately lend themselves to the use of diagrams. The formulae used by the pre-service teachers were easy to obtain and did not require too much thought or reflection.

Through metacognitive regulation, the participants were able to perform task and evaluate the processes and strategies which underpinned this performance. In doing this, they were able to provide the following insight into the reasons for using diagrams, formulae and equations to solve problems:

- Enhances logical reasoning
- Enhances confidence of the problem solver
- Provides useful starting point to solve roblems
- Enables problems to be solved more easily and speedily
- Overcomes language barriers, to some extent

4.2.3 Research question 3

Research question three was: How do pre-service Science and Mathematics teachers include the use of visualisation strategies when preparing their lessons? This question brought about two main themes from the data, they are:

- The variations to how pre-service Science and Mathematics teachers plan the use of visualisation strategies into problem solving.
- The different strategies they use in the planning of teaching problem solving using visualisation strategies.

These two themes emerged from the analysis of the lesson plans that pre-service teachers compiled. Five pre-service teachers participated in this stage of the research. Each of the pre-service teachers was asked to write out a lesson plan in their field of study, which was either in Science or Mathematics. Most of the pre-service teachers submitted two lesson plans, one in the field of Mathematics and one in the field of Science. The requirements of the lesson plan were to include visualisation methods in the teaching of whatever topic they had selected. The duration was left for them to decide. A cover page was given which asked for biographical details as well as basic information about the lesson such as: learning area, grade, duration, visual strategies used, and any general comments that they felt should be included regarding their lesson plan. In retrospect, the pre-service teachers should have been asked to provide the context in which their lesson plan was planned. This would have given a background to the lesson.

Before the analysis of the lesson plans, a brief summary of each pre-service teacher's lesson plans is provided. The pre-service teachers were allocated numbers as they submitted their questionnaire (which was step one of the data collection method). Therefore, although only five participants completed the lesson plan, the pre-service teachers' numbers in order are: Pre-service teacher 2, Pre-service teacher 3, Pre-service teacher 4, Pre-service teacher 8, and Pre-service teacher 10.

4.2.3.1 An overview of each pre-service teachers' lesson plan/s:

- **Pre-service teacher 2**

Pre-service teacher 2 did two lesson plans, one in Science and one in Mathematics. The Mathematics lesson plan spanned two hours and covered the topics of mass measuring instruments, which included reading off scales. This pre-service teacher did not provide a sequential order in which each step of the lesson would unfold. The visual strategies used were pictures of scales; a number line chart; and a bathroom scale. The visualisation strategies were used to help learners learn and practice the skill of reading off a scale. The lesson plan, although lacking details, seemed feasible in a classroom situation.

Pre-service teacher 2's Science lesson plan was of an hour's duration and covered the topic of circuit diagrams. The lesson was teacher-based and left no room for learners to discover knowledge. The visualisation strategies included charts, cut-out symbols and circuit parts. The visualisation strategies used were charts, which were used to provide learners with information; and the construction of circuits was used to cement the information that was taught. The lesson plan, although lacking details, seemed feasible in a classroom situation.

- **Pre-service teacher 3**

Pre-service teacher 3 completed one lesson plan in Mathematics for the duration of 45 minutes. The topic of the lesson plan was 'constructing the centre of the circle'. The structure was based on giving learners instructions to follow and learners carrying out these instructions. The visualisation strategies used included the use of Mathematics sets. The Mathematics sets were to be used by the learners in following instructions given by the pre-service teacher.

- **Pre-service teacher 4**

Pre-service teacher 4 submitted two lesson plans, one in Science and one in Mathematics. The Science lesson plan was one hour long and covered the topic of the 'circulatory system'. The structure was detailed, showing what was planned to be done at each step of the lesson. The visualisation strategies that were used included games, videos, role play, charts, and presentations. The visualisation strategies were used to introduce, reiterate, and cement the concepts that were taught.

Pre-service teacher 4's Mathematics lesson plan was of one hour duration and the topic was 'classifying 2D shapes'. The lesson plan was systematic, detailed, and well researched. The visualisation strategies that were used included games, videos, and a geoboard. Visualisation strategies were used for learners to discover parallel lines and to test learners on the shapes, thereby clearing any misconceptions. It appeared to be a realistic lesson plan.

- **Pre-service teacher 8**

Pre-service teacher 8 submitted two lesson plans, one in Science and the other in Mathematics. The Science lesson plan was an hour and covered the topic of DNA replication. The lesson was uncomplicated and easy to follow with research clearly having been done on the content. The lesson plan, however, seemed to lack creativity and was more teacher-orientated. Visualisation strategies used included a video using a projector and images from a textbook. The images in the textbook were used for learners to follow the DNA replication process and the video was used to reiterate what was taught.

Pre-service teacher 8's Mathematics lesson plan was an hour. The topic of the lesson plan was 'circle geometry' and 'the angle at the centre of a circle'. The lesson plan was easy to follow. Visualisation strategies used included a sketch pad (program) and cut-outs of circles. The cut-outs were used for learners to measure the angle at the centre of the circle. A sketch pad was used to explain the theorem of finding the centre of a circle. The sketch pad as a tool is most powerful as learners are able to see moving lines and angles, this makes it an interactive tool for teaching and learning.

- **Pre-service teacher 10**

Pre-service teacher 10 submitted one Science lesson plan. The topic was the theory of evolution and its duration was an hour. The structure of the lesson plan was more of an activity rather than a detailed lesson plan. The visualisation strategies used included different colour beans representing different organisms and different colour cardboard to represent the different habitats. The visualisation strategies were used for learners to simulate the process of natural selection. Learners had a hands on experience of the process while being facilitated by the pre-service teacher. Pre-service teacher 10 organised his learners into groups of five. Learners got to physically place the various coloured beans (representing various organisms) onto the various coloured cardboard (representing various habitats). The task for learners was to randomly select the beans and thereafter analyse them based on a worksheet given.

4.2.4 A comparison of the pre-service teachers' Science and Mathematics lesson plan/s

- **Visualisation strategies used in Science lesson plans:**

There were four common visualisation strategies that were found. Three pre-service teachers used pictures. Two pre-service teachers used objects as their visual aid. Two pre-service teachers used the visual aid of a video. One pre-service teacher used role play/presentation as their visual aid. It is noteworthy that although there were five pre-service teachers, in total they submitted four Science lesson plans but eight different visualisation strategies were noted. This number of visualisation strategies could be higher considering the pre-service teachers were third year pre-service teachers and only a year away from becoming teachers. It is assumed that they would be ofay with an ever greater amount of visualisation strategies to take to their own classrooms. However, the few that were used were used in an effective way as per their individual lesson plans.

- **Visualisation strategies used in the Mathematics lesson plans:**

One pre-service teacher used pictures. Another pre-service teacher used games as a visualisation strategy. More pre-service teachers used some sort of computer software as a visual aid. Three pre-service teachers used mediating artifacts of different varieties. It is significant that there was more of a variety of visualisation strategies used in the Mathematics lesson plan as compared to that of the Science lesson plans. There were four Mathematics lesson plans and eight visualisation strategies used in the four lesson plans. A reason for this could be that the pre-service teachers used the visual aids that they were the most comfortable with in their lesson plans.

- **Common visualisation strategies found in the Science and Mathematics lesson plans:**

There were two common visualisation strategies found in both the Science and Mathematics lesson plans. One was the use of pictures and the other was the use of different kinds of mediating artifacts. The fact that there were just two common visualisation strategies used in both types of lesson plans shows that the pre-service teachers considered some visualisation strategies better suited to certain subjects than others, for example, none of the pre-service teacher's used a computer program in the Science lesson plans, but two of them used it in their Mathematics lesson plan.

- **How visualisation strategies were incorporated into the lesson plans:**

There were five common ways that they incorporated the visualisation strategies into their lessons. Firstly, pictures were used to construct meaning and to test learners' understanding of a concept that was taught to them. Secondly, objects were used for learners to discover a concept and a process taught to them. Thirdly, videos were used to reiterate, cement concepts taught, and/or introduce a topic. Fourthly, games were used to test learners' knowledge and to clear misconceptions they would have had. Fifthly, computer programs were used to show learners what parallel lines are. It is significant that the visualisation strategy that involved the use of a video was the most multipurpose one used. This one strategy was only used by two pre-service teachers but served three different purposes. The planning activities of incorporating visualisation strategies to solve problems allowed pre-service teachers to think deeply about knowledge construction. Their consciousness of how students can be enabled to solve problems using visualisation strategies was raised, and this signals metacognitive regulation.

Table 9: Summary of the lesson plans pre-service teachers created

Pre-service teacher	Learning area	Duration	Topic	Visualisation strategies used	How is the VS incorporated into the teaching of problem solving?
2a	Science	2hours	Mass measuring instruments, reading scales.	Pictures of scales, number line chart, bathroom scale.	Chart given for learners to show their understanding of 'reading off a scale'.
2b	Mathematics	60mins	Circuit diagrams.	Charts, cut-out symbols, circuit parts.	Chart is used to give learners information. Construction of circuits used to cement information taught.
3	Mathematics	45mins	Constructing the centre of the circle.	Mathematics set.	Learners to use the Mathematics set instruments to aid them in constructing a circle along with instructions given by the pre-service teacher.
4a	Science	60mins	Circulatory system.	Games, video, role play, charts and presentations.	Examples were used as teaching tools to introduce, reiterate, and cement concepts.
4b	Mathematics	60mins	Classifying 2D shapes.	Games, video, ohp, geoboard.	Cardboard shape cut-outs were used for learners to discover parallel lines, which was a new concept, card game used to test learners on the shapes just learned and to clear misconceptions.
8a	Science	60mins	DNA replication.	Video of the projector and images from textbook.	Images in textbook used to follow the process, video used to reiterate the lesson.
8b	Mathematics	60mins	Circle geometry, angle at the centre of a circle.	Sketch pad, cut-outs of circles	Students to measure the angle at the centre of the circle using the cut-out, sketch pad used to explain the theorem.
10	Life Science	60mins	Theory of evolution.	Different colour beans (to represent different organisms) and different colour cardboard (to present different habitats) to simulate the process of natural selection.	The activity was facilitated by the teacher and used to simulate the process of natural selection. Students had hands on experience of the process.

Table 9 highlights two aspects, firstly, how pre-service teachers plan the use of visualisation strategies when planning a lesson. Secondly, it shows the visualisation strategies that the pre-service teachers did use. A possible interpretation of Table 9 is firstly that the pre-service teachers used visualisation strategies in their lesson plans to mainly introduce and/or discover a new concept or process. Secondly, the visualisation strategies used varied, but there was more of a variety in the Science lessons than in the Mathematics lessons.

4.3 Responses from the interviews

Two interviews were conducted a month apart. The purpose of interview one was to ask the pre-service teachers to clarify some of their responses from the various tools. Clarity was needed because some responses were not legible or comprehensible, or questions were not answered, and/or more details on certain questions were needed. Due to the variation of clarity needed from each of the pre-service teachers, all interview schedules were specific to each pre-service teacher.

Questions in interview two were intended to clarify ambiguous or superficial responses given in interview 1. Therefore, not all the pre-service teachers had the same set of questions. However, the one common question that all the participating pre-service teachers were asked was "What did you learn and/or gain from being a part of this research?" The response of one of the pre-service teachers is as follows:

- Pre-service teacher 3:

"*During first and second year this visualisation thing I didn't was some kind of oral stuff but I was more interested in what I was going to be tested on. Visualisation can help each and every one to synthesis and analyse a concept. Yes I have started thinking more about visualisation and the benefits. Even in deep rural schools I can use a variety of things like I had a lecturer last year, I think it was math's method lecture that used … maybe I want to measure…you can come into class with your own cut out protractors and a box and maybe you can give them. I think they will learn better like that rather than writing*".

Pre-service teacher 3 explained in his own words that his initial interest in visualisation was limited to which of its aspects would be tested. However, this grew to him understanding the need for visualisation and therefore the benefits thereof. This change in thinking for Pre-service teacher 3 ended with him wanting to use his newly found appreciation for visualisation with his learners.

Table 10: Summary of pre-service teachers' opinions of being a part of this study

Pre-service teacher's number	Positive feedback about being a part of study?	Have you learned more about visualisation and problem solving since the questionnaire?	Agree about benefits of using visualisation to teach problem solving?	Will use visualisation when teaching?	Will use visualisation more when solving problems?
2	✓	✓	✓	✓	✓
3	✓	✓	✓	✓	✓
4	✓	✓	✓	✓	✓
8	✓	✓	✓	✓	✓
10	✓	✓	✓	✓	✓

Table 10 displays the responses of the pre-service teachers based on their experience of being a part of the study. A possible interpretation of Table 10 is that all the participating pre-service teachers grew in their knowledge of what visualisation is, and the benefits of problem solving and teaching it.

The present chapter explored the data collected from the study. The next chapter will attempt to draw conclusions from the findings presented here.

4.4 Conclusion

In this chapter, the themes from the findings of the data generation tools were discussed. There were three research questions which the themes revolved around. Research question one was: *What do pre-service Science and Mathematics teachers understand by problem solving within a visualisation context?* Three themes arose as a result of this research question, 1.Pre-service Science and Mathematics teachers' understanding of problem solving, 2. Pre-service Science and Mathematics teachers' understanding of visualisation, 3. Pre-service Science and Mathematics teachers' understanding of the link between problem solving and visualisation.

Research question two was: *Why do pre-service Science and Mathematics teachers who use visualisation strategies choose these strategies when teaching problem solving?* In answering this question, two themes emerged from the data. 1. The different visualisation strategies that pre-service Science and Mathematics teachers use when solving problems. 2. Pre-service Science and Mathematics teachers' reasons for the use of the visualisation strategies that they used.

Research question three was: *How do pre-service Science and Mathematics teachers include the use of visualisation strategies when preparing their lessons?* Two themes emerged from this question. 1. The different ways that pre-service Science and Mathematics teachers plan the use of visualisation strategies into problem solving. 2. The variety of strategies they would use in the planning of teaching problem solving using visualisation strategies. The concluding points of the findings will be discussed in the following chapter. The following chapter will also discuss the limitations of this study, as well as recommendations for further research.

CHAPTER 5 CONCLUSION

5.1 Introduction

The previous chapter presented the analysis of the data in terms of the themes that emerged from it. This chapter will discuss: the main findings of the study; recommendations for teacher training institutes, and recommendations for further research; limitations of the study; and a final conclusion.

5.2 Main findings of research

The findings of the study were guided by the research questions, which in turn guided the themes that emerged from the data. A summary of the concluding findings will be mentioned in order of the three research questions that underpinned the study.

5.2.1 Research question one

Research question one was: *What do pre-service Science and Mathematics teachers understand by problem solving within a visualisation context?*

In terms of understanding problem solving within a visualisation context, all the participating pre-service teachers understood the terms in isolation. Furthermore, all the participating pre-service teachers found that there is a link between visualisation and problem solving. Their explanation varied, however, the underlying reason was that they understood visualisation to help solve problems with greater ease. Pre-service teachers who viewed problem solving as a process involved in finding answers, underscored the value of one's cognitive potential to solve problems, and the vital role of critical thinking in problem solving. The acquisition of skills and expertise were cited as crucial determinants to engaging in problem solving activities.

5.2.2 Research question two

Research question two was: *Why do pre-service Science and Mathematics teachers who use visualisation strategies choose these strategies when teaching problem solving?*

There seems to be a tendency for pre-service teachers to create diagrams when solving problems. Diagram usage is closely associated with the pre-service teachers' gravitation towards using formulae. Not many chose to create equations or use other algebraic methods.

When working with Science problem solving questions, the pre-service teachers preferred to use formulae rather than a combination of formulae and diagrams, and they reached the correct answer by doing so. They did it this way because they felt that formulae are adequate and accurate strategies to use. It may be that the problems selected for the Science task sheet did not adequately lend themselves to the use of diagrams. The formulae used by the pre-service teachers were easy to obtain and did not require too much thought or reflection. The use of diagrams to solve Science problems would possibly have featured more frequently had the problems been more complex. Solutions to simpler problems were linked to the use of formulae without diagrams,

Pre-service teachers' positive attitudes towards using visualisation strategies to solve problems indicated that they would use visualisation strategies to solve problems themselves, and to teach this to their learners. The importance of using visualisation strategies in under-resourced settings which lacked concrete artefacts to facilitate conceptual understanding, was emphasised by the participants. The use of visualisation strategies to understand a problem, plan how to solve it, and to solve the problem and reflect on the solution, indicate how metacognitive regulation features in learning to teach.

5.2.3 Research question three

Research question three was: *How do pre-service Science and Mathematics teachers include the use of visualisation strategies when preparing their lessons?*

The pre-service teachers used visualisation strategies in their lesson plans to mainly introduce and/or discover a new concept or process for learners. The visualisation strategies used varied, but there was more of a variety in the Science lessons than in the Mathematics lessons. Pre-service teachers believed that lessons which were planned in a way that enabled learners to "see the data" were highly effective. In thinking about how they could plan to teach, pre-service teachers thought deeply about different visualisation strategies, and included concrete artefacts, pectures, games and computer programmes, which they believed could enhance conceptual understanding.

5.3 Recommendations

The recommendations that follow (5.3.1 and 5.3.2) are recommendations based on the findings of this research.

5.3.1 Recommendations for teacher training institutes

Teacher training curricula should include a module on problem solving, and this view was shared by several participants. It should include the teaching of problem solving using visualisation as separate and combined entities. In addition to this, pre-service Science and Mathematics teachers should be taught a variety of visualisation strategies, as well as the benefits of its usage for themselves and their future learners. If pre-service teachers view themselves as experts who are skilled in using visualisation strategies to solve problems, it is likely that they would be more confident to teach problem solving in Mathematics and Science classrooms, and the potential for better learner performance can be enhanced.

5.3.2 Recommendations for further research

Section 5.4 refers to the limitations of this study. One of the limitations that will be discussed is that the study's findings cannot be generalised to other teacher training institutes because the study was only conducted in one teacher training institute. From this point, it is recommended that research into pre-service teachers' use of visualisation in a problem solving context be conducted in more teacher training institutes. More in-depth research can be conducted among practicing teachers, to examine what constrains and what enables the use of visualisation strategies to solve problems in actual school contexts. This knowledge could be used to tailor teacher training to address specific challenges related to this phenomenon.

5.4 Limitations of study

The limitations that will be discussed are the influences, shortcomings, or conditions that could not be controlled in the study. The reason for mentioning them is that they placed restrictions on the methodology and conclusions. The following limitations that will be discussed relate directly to this study: time constraints, acts of nature, generalisation, and the process of probing.

A low and slow response rate was experienced from respondents in terms of the data collection. The questionnaire had been administered at a time prior to the commencement of the pre-service teachers' exams, which in retrospect, was poor planning. This resulted in me having to make numerous pleas via email and phone to respondents for the submission of the completed tools. Considering that the study was qualitative in nature, this meant that it generally took more time to collect and analyse the data when compared to quantitative research (Anderson, 2010).

The act of nature (Verial, 2010) refers to the personal issues encountered that slowed the researcher's progress, and in one instance, halted her progress. A semester into this dissertation, I, as the researcher was diagnosed with spondylosis of the spine and underwent treatment. This resulted in an interruption to the research process.

Application of the findings of this study is limited to the one teacher training institute where this study was conducted. However, it was not intended to generalise the findings of this study to other teacher training institutes.

The final limitation of this study as the researcher's lack of probing during the interviews with the interviewees. This issue emerged while analysing the data and presenting the findings of this study. Perhaps it was due to a lack of experience as an interviewer on my part. However, it was found that more data could have been gathered had some of the interviewee's responses been further probed. Anderson (2010) explains that "[r]esearch quality is heavily dependent on the individual skills of the researcher and more easily influenced by the researcher's personal biases and idiosyncrasies" (p.6). I tried to curb my bias by keeping a reflective attitude with a reminder that no tool is more superior than another, and to allow the data to 'speak' for itself. In addition to this, many qualitative data generation tools were used to make the data process more valid.

5.5 Conclusion

This chapter discussed the main findings of this study (relating to each research question), the recommendations made from the findings (which were directed at teacher training institutes and other researchers), as well as the limitations of the study. This study looked at pre-service Science and Mathematics teachers' use of visualisation in teaching problem solving to learners. Problem solving is emphasised in this study as a vital aspect of education and curriculum, as it prepares learners for real world problems.

6 REFERENCES

Ainsworth, S.E., Bibby, P.A , and Wood, D.J. (1997). Information technology and multiple representations: new opportunities-new problems. *Technology, Pedagogy and Education Publication details, 6* (1), 93-105.

Anderson, C. (2010). Presenting and Evaluating Qualitative Research. *American Journal of Pharmacy Education, 74*(8), 1-7.

Angeli, C., & Valanides, N. (2004). Examining the effects of Text-Only and Text-and-Visual Instructional Materials on the Achievement of Field-Independent Learners during Problem solving with Modelling Software. *Educational Technology Research and Development, 52*(4), 23-36.

Arcavi, A. (2003). The role of visual representations in the learning of Mathematics. *Educational Studies in Mathematics, 52*, 215-241.

Armstrong, T. (2008). Multiple Intelligences. 1-3. Retrieved on 24/06/2011 at 12:06 from

Berg, B. (2007). *Qualitative Research Methods for the Social Science s* (6th ed). Boston: Pearson Education.

Charoenruk, D. (2007). Communication Research Methodologies: Qualitative and Quantitative Methodology. In D. Doungkamol (Ed.), *Communication Research Methodologies,* 1-17.

Cohen, L., & Manion, L. (Ed.) (1994). *Research methods in education* (4th ed.). London: Routledge.

Collins. (Ed.) (2009) *Collins 30th Anniversary Edition, English Dictionary* (10th ed.). Glasgow: HarperCollins Publishers Limited.

Collinson, J.A.C. (2012). Interpretivist Research Paradigm 1-2. Retrieved on 24/06/2013 from http:://www.interpretivistresearchparadigm_jacuelynallencollinson

Copeland, R.W. (Ed.). (1979). *How Children Learn Mathematics: Teaching Implications of Piaget's Research.* (Third ed.). New York: Macmillian Publishing Co., Inc.

Cram, F. (2013). Method or Methodology, what's the difference? 1-3. Accessed on 24/06/2012 from http://whanauoraresearch.co.nz/news/method-or-methodology-whats-the-difference/

Crespo, S. (2003). Learning to Pose Mathematical Problems: Exploring Changes in Pre-service Teachers' Practices. JSTOR Retrieved 01/06/2011, from Springer http://www.jstor.org/stable/3483016

Creswell, J.W. (2003). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches*. Thousand Oaks, CA: Sage Publications.

Crossman, A. (2014). Purposive Sample. 1-3. Accessed on 20/07/2014 from http://sociology.about.com/od/Types-of-Samples/a/Purposive-Sample.htm

Data Analysis Australia: Response Rates. (2014). *Strategic information consultants*, 1-4. Retrieved on 20/07/2014 from http://www.daa.com.au/analytical-ideas/response-rates/

de Chadarevian, S., & Hopwood, N. (Ed.). (2004). *Models: The Third Dimension of Science* . Stanford, California: Stanford University Press.

Department of Basic Education (DoBE) (2011a) DoBE. *Curriculum Assessment and Policy Statement (CAPS) Mathematics*. Government Printers, Pretoria.

Department of Basic Education (DoBE) (2011b) DoBE. *Curriculum Assessment and Policy Statement (CAPS) Natural Science*. Government Printers, Pretoria.

Department of Education (DoE) (2002) *Revised National Curriculum Statement (RNCS).* Government Printers, Pretoria.

Dey, E.L. (1997). Working with Low Survey Response Rates: The Efficacy of Weighting Adjustments. *Research in Higher Education, 38*(2), 215-227. doi:10.1023/A:1024985704202

Department of Basic Education. (2012). National Senior Certificate Examination School's Subject Report (pp. 1-274). KwaZulu Natal.

Dogru, M. (2008). The Application of Problem Solving Method on Science Teacher Trainees on the Solution of the Environmental Problems. *Journal of Environmental & Science Education, 3*(1), 9-18.

Elliott, S. (2001). Visualisation and Using Technology in a Level Mathematics. *BSRLM*, 18(1&2), 45-50.

Fischer, F., Bruhn, J., Gräsel, C., & Mandl, H. (2002). Fostering collaborating knowledge construction with visualisation tools. *Learning and Instruction, 12*, 213-232.

Flavell, J.H. (1979). Metacognition and cognitive monitoring: A new area of cognitive developmental inquiry. *American Psychologist*, 34(10), 906-911.

Friedlander, A., & Tabach, M. (2001). Promoting multiple representations in algebra. In A. A. Cuoco (Ed.), *Yearbook of the National Council of the Teachers of Mathematics: The Roles of Representation in School Mathematics* (pp. 173-185). Israel, Reston, Virginia: The Council.

Gilbert, J.K. (2004). Models and modelling: Routes to more authentic Science education. *International Journal of Science and Mathematics Education, 2*, 115-130.

Gilbert, J.K., & Justi, R.S. (2010). Modelling, teachers' views on the nature of modelling, and implications for the education of modellers. . *International Journal of Science Education, 24*(4), 369-387.

Golafshani, N. (2003). Understanding Reliability and Validity in Qualitative Research *The Qualitative Report* (Vol. 8, pp. 597-607).

Gorland, C. (2014). Paradigms as a Framework for Scientific Research. *The Impact of Paradigms on Scientific Knowledge*, 1-2. Retrieved from https://bu.digication.com/chase_gorland_boston_university/The_Impact_of_Paradigms_on_Scientific_Knowledge

Gouws, F.E. (2007). Teaching and learning through multiple intelligences in the outcomes-based education classroom. *Africa Education Review, 4*(2), 60-74.

Hardin, L.E. (2002). Problem solving concepts and theories. *JVME, 30*(3), 227-230.

Ho, S.Y. (2009). Seeing the Value of Visualisation. 1-4. Retrieved from http://www.valuevisualisation.com

Hobden, P.A. (2002). Reclaiming the meaning of problem solving: The need for a common understanding of the terms problem and problem solving. In C. Lubisi & Malcolm (Eds.), Proceedings of Annual meeting of South African Association of Research in Mathematics, Science and Technology Education (pp. 221-245). Durban: SAARMSTE.

Holbrook, A.L., Krosnick, J.A., & Pfent, A. (2007). The Causes and Consequences of Response Rates in Surveys by the News Media and Government Contractor Survey Research Firms. In C.B. Lepkowski J.M., J.M. Tucker, E. de Leeuw, L. Japec, P.J. Lavrakas, M.W. Link, & R.L. Sangster (Eds), *Advances in Telephone Survey Methodology* (pp. 499-678). Chicago: John Wiley & Sons, Inc.

http://gse.buffalo.edu/fas/shuell/cep564/metacog.htm

http://www.ssc.wisc.edu/~oliver/SOC357/Lectures%20and%20Notes/SamplingBigSlides.pd http://www.thomasarmstrong.com/multipleintelligences.php

Hwang, W.Y., Su, J.H., Huang, Y.M., & Dong, J.J. (2009). A Study of Multi-Representation of Geometry Problem Solving with Virtual Manipulatives and Whiteboard System. *Educational Technology & Society, 12*(3), 229-247.

Johnson, B., & Christensen, L. (2008). Educational research: Quantitative, qualitative, and mixed approaches. 1-37.

Joppe, M. (2000). The Research Process. Retrieved from http://www.ryerson.ca/~mjoppe/rp.htm.

Laerd, D. (2012). Purposive sampling. 1-4. Accessed from http://dissertation.laerd.com/purposive-sampling.php

Lai, E.R. (2011). Metacognition: A Literature Review *Always Learning* (pp. 1-41).

Lengler, R., & Eppler,M.J. (2007). A periodic table of data visualisation methods. http://www.visual-literacy.org/periodic_table/periodic_table.html

Lengler, R.., & Eppler, M.J. (2006). Towards A Periodic Table of Visualisation Methods for Management. 1-6. Retrieved from http://www.towards_a_periodictable_of_visualisationmethods_for.management//article

Lewis, A., Smith, D. (1993). Defining Higher Order Thinking theory into practice 32 (3). Ohio: College of Education.

Livingston, J.A. (1997). Metacognition: An Overview. 1-5.

Lowrie, T., & Kay, R. (2001). Relationship between Visual and Nonvisual Solution Methods and Difficulty in Elementary Mathematics. *The Journal of Educational Research, 94*(4), 248-255.

Mackenzie, N., & Knipe, S. (2006). Research dilemmas: Paradigms, methods and methodology. *Issues in Educational Research, 16*, 1-11.

Miller, A.L., & Dumford, A.D. (2014). Open-Ended Survey Questions: Item Nonresponse Nightmare or Qualitative Data Dream? *Survey Practice, 7*(5), 1-11.

Motshekga, A. (2012). Statement on the release of Annual National Assessments Results for 2012 Retrieved 24/09/2013, 2013, from http://www.info.gov.za/speech/DynamicAction?pageid=461&tid=92811

Mourtos, N.J., Okamoto, N.D., Rhee, J. (2004). Defining, teaching, and assessing problem solving skills. *7th UICEE Annual Conference on Engineering Education*, 1-5. https://ae.sjsu.edu/files/public/nikos/backup/pdf/UICEE%2004%20Mumbai.pdf

Oliver, S. (Producer). (2002, 18/12/2014). *Sampling*. Retrieved from

Ozel, S., Capraro, R.M., & Yetkiner, E.Z. (2008). A technology intervention using multiple representations on Mathematics. 1-7.

Özmantar, M.F., Akkoç, H., Bingölbali, E., Demir, S., & Ergene, B. (2010). Pre-Service Mathematics Teachers' Use of Multiple Representations in Technology-Rich Environments. *Eurasia Journal of Mathematics, Science & Technology Education, 6*(1), 19-36.

Palys, T. (2009). Purposive sampling. 1-3. http://www.sfu.ca/~palys/Purposive%20sampling.pdf

Pienaar, H.C., Nieman, M.M., & Kamper, G.D. (2011). Implementing a teaching approach based on multiple intelligence theory in a South African school: A case study. *African Education Review, 8*(2), 267-285.

Price, M. (2013). Convenience Samples: What they are, and what they should (and should not) be used for. 1-4. Retrieved from https://hrdag.org/convenience-samples-what-they-are/

Rahim, M.H., & Siddo, R. (2009). *The use of visualisation for learning and teaching Mathematics* (pp. 496-500).

Rau, M.A., Aleven, V., & Rummel, N. (2009). *Intelligent Tutoring Systems with Multiple Representations and Self-Explanation Prompts Support Learning of Fractions*. Amsterdam: Netherlands: IOS Press.

Reddy, V. (2012). *SA and Honduras at bottom of TIMSS rankings - HSRC* (pp. 1-3). Centurion.

Roszkowski, M.J., & Bean, A.G. (1990). Believe it or not! Longer questionnaires have lower response rates. *Journal of business and psychology, 4*(4), 495-509.

Schraw, G., & Moshman, D. (1995). Metacognitive Theories. *Educational Psychology Review, 7*(4), 351–371.

Schwartz, N.H. (Producer). (2010). Metacognition Theoretical Frameworks to Understanding Learning. *Psych 605 Advanced Human Learning*. [Education] Retrieved from http://www.Metacognition1Final.Fall_2010

Department of Basic Education (DoBE) (2011a) DoBE. *Curriculum Assessment and Policy Statement (CAPS) Natural Science* . Government Printers, Pretoria.

Department of Education (DoE) (2002) Revised National Curriculum Statement (RNCS). Government Printers, Pretoria.

Sivo, S.A., Saunders, C., Chang, Q., Jiang, J. (2006). How Low Should You Go? Low Response Rates and the Validity of Inference in IS Questionnaire Research. *Journal of the Association for Information Systems, 7*(6), 352 351-414.

Smith, M.K. (2007). Howard Gardner and multiple intelligences', the encyclopaedia of informal education 1-10. Retrieved from http://www.infed.org/thinkers/gardner.htm.

Tewksbury, R. (2009). Qualitative versus Quantitative Methods: Understanding Why Qualitative Methods are Superior for Criminology and Criminal Justice. *Journal of Theoretical and Philosophical Criminology, 1*(1), 1-21.

Tongco, D.C. (2007). Purposive Sampling as a Tool for Informant Selection. *Ethnobotany Research & Applications*, *5*(1), 147-158.

Trochim, W.M.K. (2006). Nonprobability Sampling. 1-4. Retrieved from http://www.socialresearchmethods.net/kb/sampnon.php

van Kraayenoord, C.E. (2010). The Role of Metacognition in Reading Comprehension. 277-303. Retrieved from http://www.i4.psychologie.uni-wuerzburg.de/fileadmin/06020400/user_upload/Lenhard/Krajenord2010.pdf

Veenman, M.V.J., Hout-Wolters, B.H.A.M, Afflerbach, P. (2005). Metacognition and learning: conceptual and methodological considerations. *Springer*, *1*(1), 3-14. doi: DOI 10.1007/s11409-006-6893-0

Verial, D. (2010). Examples of Limitations of a Study. 1-5. Retrieved from http://classroom.synonym.com/examples-limitations-study-14461.html

Wahyuni, D. (2012). The Research Design Maze: Understanding Paradigms, Cases, Methods and Methodologies. *JAMAR*, *10*(1), 69-80.

Wheatley, G. H. (1995). Problem Solving from a Contructivist Perspective. In D. R. Lavoie (Ed.), *Toward a cognitive-Science prespective for scientific problem solving: A monograph of the National Association for Research on Science Teaching, Number Six*. Manhattan: KS: Ag Press.

Williams, J. (2011). Research Paradigm and Philosophy. *Dissertation Writing Tips*, 1-2. Retrieved from http://www.howtodo.dissertationhelpservice.com/research-paradigm-and-philosophy

Wilson, T, V. (2006) How the Earth Works. Retrieved on 19/02/2015 from HowStuffWorks.com. http://Science .howstuffworks.com/environmental/earth/geophysics/earth.htm

Yilmaz, R., Argun, Z., Keskin, M.O. (2009). What is the Role of visualisation in Generalization Processes: The Case of Pre-service Secondary Mathematics Teachers. *Humanity & Social Science s Journal*, *4*(1), 130-137.

APPENDIX A

Questionnaire

Researcher	: L. Govender
Title of research:	An Exploration of pre-service Mathematics and Science teachers' use of visualisation techniques for solving problems: A case study at a South African University.

Name and Surname: Contact Number:

Student number:

Section A

1. Name and Surname: ____
2. Student number: _________________________________
3. Age : _________________
4. Phase : _________________
5. Learning areas : _________________________________
6. Race : ______________________________

Section B

1.1 Have you worked with problem solving?

____

1.2 How many problems have you worked out this week?

____

____

1.3 Do you enjoy working with problem solving?

____

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1.4 How many minutes does it usually take you to solve a problem?

____

____

1.5 What does problem solving mean?

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1.6 How does problem solving help you?

____

____

1.7 Do you think it is important to teach learners how to solve problems?

____

____

1.8 What skills does problem solving develop?

____

____

1.9 What kinds of problem solving strategies are you aware of?

____

____

1.10 How would you sum up what problem solving is?

____

____

2.1 Have you worked with visualisation strategies? When and how?

____

____

2.2 Do you enjoy working with visualisation strategies? Why?

____

____

2.3 How many minutes does it usually take you to solve a problem using visualisation strategies as compared to algebraic means?

____

____

2.4 What does visualisation mean to you?

____

____

2.5 How does visualisation help you to solve problems?

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____

2.6 Do you think it is important to teach learners how to visualise (use visualisation strategies)? Explain.

____

____

2.7 List some of the visualisation strategies that can be used to solve problems.

____

____

2.8 Which skills have you used most frequently?

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Section C

1.1 Do you think there is a link between problem solving and the use of visualisation strategies?

____

1.2 What is the link?

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____

1.3 Give an example of how visualisation and problem solving can work together?

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____

____

Section D

1.1 Do you have any comments you would like to make regarding problem solving in a visualisation context? If so, please feel free to add them here:

____

____

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APPENDIX B

Mathematics problem solving worksheet

Name and Surname: ___________________________________

Cell number : ________________________

Solve the following problems. Show all working out as well as you reasons for your working out.

1.1 Nandi wants to tie a ribbon around the sides of a hexagonal box and then tie a bow. Each side of the box is 4cm long. She needs 14cm for the bow. Is half a metre of ribbon enough? Explain. Then state exactly how long the ribbon must be.

1.2 I chose this method and strategy because:

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2.1 The length of a rope, to which a cow is tied, is increased from 19m to 30m. How much additional ground will it be able to graze in? Assume that the cow is able to move on all sides with equal ease.

2.2 I chose this method and strategy because:

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3.1 Find the equation of the tangent at (0 , 2) to the circle with equation

$$(x + 2)^2 + (y + 1)^2 = 13$$

3.2 I chose this method and strategy because:

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4.1 The triangle bounded by the lines $y = 0$, $y = 2x$ and $y = -0.5x + k$, with k positive, is equal to 80 square units. Find k.

4.2 I chose this method and strategy because:

____

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APPENDIX C

<u>Science Task sheet</u>

Name and Surname:

Student number:

Solve the following problems. Show all working out as well as you reasons for your working out.

1.1 A piece of wood that measures 3.0 cm by 6.0 cm by 4.0 cm has a mass of 80.0 grams. What is the density of the wood? Would the piece of wood float in water? (volume = L x W x H)

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1.2 I chose this method and visual strategy/ies because:

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2.1 A cup of gold coloured metal beads was measured to have a mass 425 grams. By water displacement, the volume of the beads was calculated to be 48.0 cm3. Given the following densities, identify the metal.

Gold: 19.3 g/mL   Copper: 8.86 g/mL   Bronze: 9.87 g/mL

2.2 I chose this method and visual strategy/ies because:

3.1 A little aluminium boat (mass of 14.50 g) has a volume of 450.00 cm3. The boat is place in a small pool of water and carefully filled with pennies. If each penny has a mass of 2.50 g, how many pennies can be added to the boat before it sinks?

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3.2 I chose this method and visual strategy/ies because:

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APPENDIX D

<u>Lesson plan task</u>

Researcher : L. Govender

Title of research: An Exploration of pre-service Mathematics and Science teachers' use of visualisation techniques for solving problems: A case study at a South African University.

Name and Surname : ____

Contact Number : ____

Learning area : ____

Grade : ____

Duration of lesson : ____

Topic : ____

Visual strategies used : ____

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General comments :

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[Participants please note that this is the cover page for your lesson plan.]

APPENDIX E

Ethical Cleareance

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

1 April 2015

Mrs Levashnee Govender 205510259
School of Education
Edgewood Campus

Dear Mrs Govender

Protocol reference number: HSS/0194/015M
Project title: An experienceof pre-service mathematics and Science teachers' use of visual techniques when solving problems: A Case Study at a South African University

Full Approval – Expedited Application

In response to your application received on 11 March 2015, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol have been granted **FULL APPROVAL.**

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number.

PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the data of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....

Dr Shenuka Singh (Chair)
Humanities & Social Sciences Research Ethics Committee

/pm

Cc Supervisor: Dr R Mudaly & Dr V Mudaly
Cc Academic Leader Research: Professor P Morojele
Cc School Administrator: Ms T Khumalo

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APPENDIX F

Certificate of Editing

Exclamation Translations

To whom it may concern

The thesis entitled "An Exploration of Pre-service Science and Mathematics Teachers' Use of Visualisation in a Problem Solving Context: A Case Study at a South African University" was thoroughly edited and proofread as of 01 March 2015. I verify that it is ready for publication and/or public viewing as it is up to the expected standard.

Please take note that Exclamation Translations takes no responsibility for any content added to the document after the issuing of this certificate.

Kind regards

Melissa Labuschagne

Melissa Labuschagne trading as Exclamation Translations.

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