

**AN EXPLORATION OF THE USE OF SELECTED
CONCRETE TEACHING AND LEARNING MATERIALS
IN DEVELOPING MATHEMATICS PROFICIENCY IN
FRACTIONS IN THREE GRADE 7 CLASSES IN THE
PINETOWN DISTRICT:**

By

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ABSTRACT

This study was motivated by my concern about the poor performance of primary school learners in the area of fractions. Fraction concepts should be well established by Grade 7 but I had found that they were problematic and wished to deepen conceptual understanding by the use of concrete teaching and learning materials such as, Cuisenaire rods, number lines, fraction circles, fraction strips and beans.

The key research questions were: Can selected concrete teaching and learning materials help in developing proficiency in fractions in Grade 7 learners? Which pedagogical practices and strategies are helpful in developing mathematics proficiency in fractions?

The research methodology can be described as action research. A set of activities developed by the Rational Number Project (RNP) in 2006 was used to teach a two week programme on basic fraction concepts to three successive Grade 7 classes. After the first two completed two week cycles, the learner work was studied, input from colleagues was sought and then after reflection, the next cycle was begun, implementing the lessons learnt. The data corpus includes biographical details of each of the learners, their final Grade 7 Mathematics and English marks, achievement on a fractions test, the completed activities and focus group interview transcripts.

The theoretical framework of the RNP is based on the idea of multi-representations and in particular the translation model proposed by Lesh, one of the project researchers. In this study, the notion of mathematical proficiency is also used to analyse the different aspects of learning demonstrated by the learners.

It was found that the achievements of the third cohort of learners were higher in the final test thus indicating that perhaps the improved teaching had an effect. Analysis of the learner work indicates that while the learners can speak correctly about the fraction ideas, and even draw correct pictures, they struggle to use the mathematical symbols correctly.

PREFACE

The work described in this thesis was carried out in the School of Education, University of KwaZulu-Natal, from *January 2009* to *December 2013* under the supervision of Dr Sally Hobden (Supervisor).

This study represents original work by the author and has not otherwise been submitted in any form for any degree or diploma to any tertiary institution. Where use has been made of the work of others, it is duly acknowledged in the text.

Welcome Siyabonga Nyathi

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This has been truly an interesting journey and I trust you find it worthwhile.

Welcome Siyabonga Nyathi

DEDICATION

This thesis is dedicated to my parents, Nomusa my mother and Vusumuzi my father and children and grand - children.

Your investment in me was that you educated me,
Those seeds are bearing fruits,
I am forever grateful to you.

CHAPTER 1 INTRODUCTION

The purpose of the study was to research and improve my own practice of teaching fractions. The issue here was that the concept of fractions was not readily understood by learners in Grade 7. This work was remedial, to re-teach fraction concepts using selected concrete teaching and learning materials, to make the lessons as concrete as possible and to enhance concept development. A significant obstacle to understanding fractions is that learners think that a fraction is two different numbers when they see the numerator and the denominator of the same fraction. They confuse fractions with whole numbers.

1.1 BACKGROUND AND CONTEXT

The study was conducted at a primary school where I was a Grade 7 mathematics teacher. This school is situated in the Pinetown district of KwaZulu-Natal, South Africa. It had 1020 learners, from Grade 0 to Grade 7 all of whom are Africans. The school had twenty-five classrooms and twenty-seven educators, one principal, one deputy principal, four heads of department and nineteen level one educators. Each learner paid school fees of R150 a year which means that funds for extra resources were limited. At the time of my research I was teaching in this school and was a senior teacher. There were three Grade 7 mixed ability classes and my research was conducted with these classes.

1.2 MOTIVATION OF THE STUDY

My ex-learners often visited me with fraction problems when they were doing Grades 8, 9 and 10. These problems led me to reflect on my mathematics teaching, and to investigate the Mathematics and English Language results in the school.

Fractions are well known to be difficult for learners to master, to the extent that De Turck, (2008) argues that they should be removed from the curriculum, because they are as obsolete as Roman numerals. He maintains that learners in the primary school are not ready to grasp the concept of fractions because it is too abstract. In his opinion the problems learners experience are due to the methods used by teachers such as emphasizing algorithmic procedures (e.g. invert and multiply) without any understanding by learners of why this is

appropriate. He makes a strong case for using concrete teaching and learning materials such as decimal devices, measuring jugs, weighing scales and tape measures when teaching this concept.

His view is supported by Wu (2005) who argues that rational numbers are difficult to teach at primary school, because the learners do not yet possess the conceptual sophistication needed to work knowingly with the concept of fractions. Learning fractions is also important because it is the first excursion into abstraction. To learn the concept of fractions, learners need a clear and accurate definition of terms. They need to be able to regard fractions as numbers, and to be fluent in fraction arithmetic (Wu, 2005).

One day I went to the school library to look for a Secondary Mathematics textbook “Just Mathematics” Grade 10 (Fitton, De Jager & Blake, 1996). I paged through the book to learn about the algebra topics. To my great surprise I realised that many of the topics have their roots in fraction concepts, for example combining like terms, a strategy used in the addition and subtraction of fractions. The importance of a thorough understanding of fractions for the development of further mathematic proficiency is illustrated by looking at other aspects of mathematics. The entire study of linear equations is dependent on the slope of a line, a fraction representing the rate of change. Solving systems of linear equations is dependent on the ability to form equivalent equations and manipulate fractions, which often are part of the solution. To solve rational expressions it is necessary to apply generalized fraction concepts. Solving quadratic equations by completing the square also requires fluency with fraction manipulation. Suh (2007) describing her grade five class in which learners report that they were showing some signs of mathematics avoidance and phobia. The learners were not confident to reason through problems or apply strategies and also doubted their computational skills. I shared these experiences and the desire to address them and became motivation for this dissertation.

Based on my experiences and a preliminary reading of the literature I decided to investigate the concept of fractions in this study entitled “An exploration of the use of selected concrete teaching and learning materials in developing mathematics proficiency in fractions in three Grade 7 classes in the Pinetown district”. This was a ‘hands on’ teaching-learning experience with fractions using Rational Number Project (RNP) activities (Rational Number Project, 2010). RNP is an on-going research project, begun in 1979. It advocates teaching fractions using a model that emphasises multiple representations and connections between these different representations and has published classroom materials that use this approach. Concrete classroom teaching and learning materials used in the RNP work

included, paper folding, fraction strips, number-lines and fraction circles. Using concrete teaching and learning materials makes the lessons more active and provides an effective way for the learners to represent their thinking

Following the dictionary definition of a concrete object as “a real physical object” (Sinclair, 2003), this term will be used in this report to include what are sometimes referred to in teaching aids or manipulatives.

Concrete teaching and learning materials provide teachers with more opportunities to understand what learners are thinking by observing what they are doing with them and providing indications of how they are constructing meaning for themselves. Using concrete teaching and learning materials to solve RNP fraction activities allowed the learners to construct their own knowledge about fraction concepts. The role of concrete teaching and learning materials such as Cuisenaire rods, beans, fraction strips, number line, paper-folding and fraction circles played a pivotal role in this study.

Behr, Lesh, Post and Silver (1983) working in the RNP, provided learners with experiences with multiple models of fraction representations such as rectangular and circular shapes. They recommended that quality fraction instruction should also include increased opportunities for learner discourse and activities which allow learners to manipulate concrete teaching and learning materials and those circular and rectangular pieces, coloured beans and should be key components of instruction (Behr et al.,1983). They confirmed that concrete teaching and learning materials play an important role in the development of learners’ conceptual understanding.

Mc Neil and Jarvin (2004) concurred that the use of concrete teaching and learning materials is crucial in developing learner’s understanding of fraction ideas. They explained that concrete teaching and learning materials helps learners to construct mental referents that may help them to perform fraction tasks meaningfully and that they act as visual representations of ideas that help learners to know and understand fractions. They also enhance the abilities of learners at all levels to reason and communicate. Working with concrete teaching and learning materials deepens the understanding of concepts and relationships, makes skill practice meaningful, and leads to retention and application of information in new problem-solving situations (Mc Neil & Jarvin, 2004). They also found that the use of concrete teaching and learning materials improved learners’ and attitudes towards mathematics in general. They recommend that concrete teaching and learning materials should be central to the teaching and learning of fractions, and not for fun or to add

variety to mathematics classrooms. They should be used to emphasize hands on learning, and assist learners to construct their own knowledge of mathematics.

My own experiences and the literature on the importance of fractions, and the value of using concrete teaching and learning materials, informed the formulation of these two research questions:

1. Can selected concrete teaching and learning materials help in developing proficiency in fractions in Grade 7 learners?
2. Which pedagogical practices and strategies are helpful in developing mathematical proficiency in fractions?

1.3 RELEVANCE OF THE STUDY

The findings of this study could benefit education in South Africa. I believe that the understanding of fractions may assist in the more effective introduction of algebra in higher grades, and could lay a firm foundation for mathematics in high schools. Teachers can draw information on the use of concrete teaching and learning materials from this study, and apply it in their own classrooms.

On a practical level the findings support the case for more expenditure on concrete teaching and learning materials. Funding is a contested area in schools and strong motivations are often necessary for additional teaching and learning materials. The value of less abstract and more hands-on learning of challenging topics indicated by this study will help to make a strong case for this expenditure. This value could be extended to other learning areas such as science and geography where similar emphasis is required.

1.4 SUMMARY

The background, context and motivation for the study, the research questions and the relevance of the study have been dealt with in this chapter. In Chapter Two the literature review will be presented.

CHAPTER 2 LITERATURE REVIEW

In this chapter I will situate my study within the existing understanding of learning by providing a literature-based theoretical framework with which data analysis and interpretation can be approached. This begins with an overview of the view that I have taken of learning. I then discuss the ideas of mathematical proficiency and the Lesh translation model which will be used as frameworks in this study. Particular issues in learning fractions are then discussed, including their place in the school curriculum. The next important idea in this study is the use of concrete teaching and learning materials and these are discussed together with advantages and problems associated with their use. Language plays a role in learning, especially in this study where the language of instruction was not the home language of the learners, and so language issues are discussed.

2.1 LEARNING MATHEMATICS

Effective teaching of mathematics rests heavily on considerations about how pupils learn mathematics (Reys, Suydam & Lindquist, 1992). Behaviourism and constructivism are the two main ones. Behaviourism, a popular theory in the 1950's and 1960's and still the framework of much skills development, is based on the view that knowledge is transferred, based on a change through controlled stimulus/response conditioning, for example rote learning or copying a master craftsman. The learner is dependent upon an instructor for acquisition of knowledge. Well known behaviourists such as Thorndike, Skinner and Gagne (cited in Reys, Suydam & Lindquist, 1992) believed that learners are passive recipients of knowledge, empty vessels to be filled or clean slates (*tabula rasa*) on which the teacher can write. Behaviourists believe that knowledge is absorbed from experience, from sensory inputs, and that the purpose of education is for the learners to adopt the teacher's knowledge frameworks by drill, practice and repetition.

Constructivism is a theory (Olivier, 1992) which emphasises that the cognitive frameworks, or schema, that a learner brings to a learning situation determine what he is able to learn from it. Knowledge does not simply arise from experience rather it arises from the interaction between experience and current knowledge structures. The learner is therefore not seen as passively receiving knowledge from the environment, but that learning happens through the process known as the construction of knowledge. The construction of knowledge

involves the interaction between a learner's existing ideas and new ideas: that is new ideas are interpreted and understood in the light of that learner's own current knowledge built up out of his previous experience. Learners do not only interpret knowledge, but they organise and structure this knowledge into large units of interrelated concepts called a schema. Such schemas are valuable intellectual tools stored in memory which can be retrieved and utilised. Learning basically involves the interaction between a learner's schemas and new ideas (Olivier, 1992).

A major feature of a constructivist classroom is socializing, where learners work in groups to defend and develop their ideas. The learners are asked guiding questions and they work together to acquire the new information. The learning takes place through retrieving prior knowledge and integrating it with the new experiences it to acquire new knowledge (Olivier, 1992).

The RNP is based on constructivist principles since the learners discuss possible solutions to activities and are in charge of their own learning. The teacher moves from group to group assisting in the discussion and asking relevant questions, he serves as a facilitator. RNP researchers Lesh, Post and Behr (1987) quoted four famous contributors to constructivism are Piaget, Vygotsky, Bruner and Dienes. Piaget who is known for believing that learning happens through a process of adaptation or accommodation of information into ones schema no matter the age of a learner. Vygotsky developed the idea of a zone of proximal development, which is explained as the difference between the actual developmental level of the learner and their potential for development through problem solving and discussions with more capable learners. Bruner is known for his emphasis on the idea that learning is an active process and so RNP classroom activities are of the hands-on-minds-on type. Dienes championed the use of collaborative group work and the use of concrete teaching and learning materials, games, stories and dance in mathematics classrooms to aid understanding. Dienes' principles influenced the design of some items in the RNP activities.

My experiences of teaching mathematics has led me to believe that learners can create their own mathematical knowledge by using concrete teaching and learning materials and consequently this study is framed by a constructivist approach. This fits with the constructivist approach. This view is supported by my own experiences when teaching Mathematics in the classroom.

2.2 MATHEMATICAL PROFICIENCY

Kilpatrick (2001) in a book called *'Adding it up'* suggests that mathematical proficiency should be viewed as consisting of five interwoven strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition.

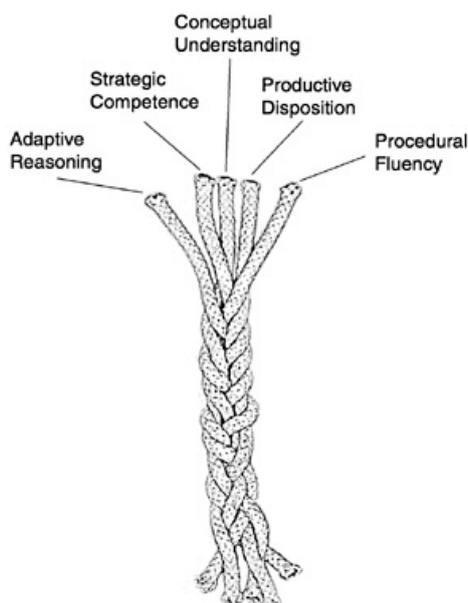


Figure 2.1 Strands of mathematical proficiency (Kilpatrick et al., 2001, p. 117)

These strands are intertwined and interdependent in the development of mathematical proficiency. The author believes that having competence in strands allows learners to connect concepts and use their understandings in future problem solving. Developing these interrelated strands promotes retention and fluency which are more powerful than rote learning. Lejeune (2011) gives the following explanations of these terms:

Conceptual understanding is defined as comprehension of mathematical concepts, operations and relations. It is more than knowing isolated facts and methods. It includes the ability to represent a mathematical situation in more than one way and to connect that understanding to the development of other mathematical procedures. Learners with conceptual understanding are better able to remember procedures and make fewer critical errors in solving problems.

Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently and appropriately. With practice accuracy and efficiency can be improved and this

helps maintain fluency. Learners need to be able to work efficiently and accurately with mathematical procedures to free up mental space to deal with new ideas. Learners who are fluent in skills and procedures do not waste time on routine tasks and can devote their attention to conceptual understanding of the topic.

Strategic competence is defined as the ability to formulate, represent and solve mathematical problems. Strategic competence relates to problem solving. Learners with strategic competence can form mental representations, detect relationships to previously known concepts and create a model solution for a problem. These learners are able to flexibly choose a method for solving problems depending on the demands of the problem.

Adaptive reasoning is defined as the capacity for logical thought, reflection, explanation and justification. Learners who possess adaptive reasoning can justify, or give sufficient evidence why their answers are correct. Adaptive reasoning allows learners to decide logically whether a solution method is appropriate for the problem they are trying to solve.

Productive disposition is defined as the habitual inclination to see mathematics as sensible, useful and worthwhile coupled with a belief in diligence and in one's own efficacy. Learners need to believe that they are capable of figuring out problems and that these problems are not arbitrary. Learners with a productive disposition are more likely to be confident in their abilities and knowledge. Learners are more likely to develop other strands of proficiency if they have productive disposition.

2.3 THE LESH TRANSLATION MODEL

This well-known model was suggested by Richard Lesh, a director of the Rational Number Project (Lesh, Cramer, Doerr, Post & Zawojewski, 2003). He suggested in this model that mathematical ideas could be represented in five ways; written, verbal, pictorial, by models and in real life situations. Learners learn by having opportunities to explore ideas in these different ways and by making connections between the different representations. This model guided the development of the RNP curriculum (Lesh et al., 2003).

Lesh (2006) also used a basket-ball analogy to explain the instruction needed by learners. In preparing for a game the coach provides a sensible mix of practice in the basics or fundamentals then provides a more authentic practice for the complexity of a real game. In the same way, we need to know what computations learners can do and what kind of situations (or

systems) they can describe (or interpret) through mathematical models. A sensible mix of learning and practice in mathematical computation and application of mathematical (or other problem-solving) models should be the goal of teachers for their learners. “The constructs we used to make sense of the world are also the constructs used to mould and shape the world” (Lesh, 2006).

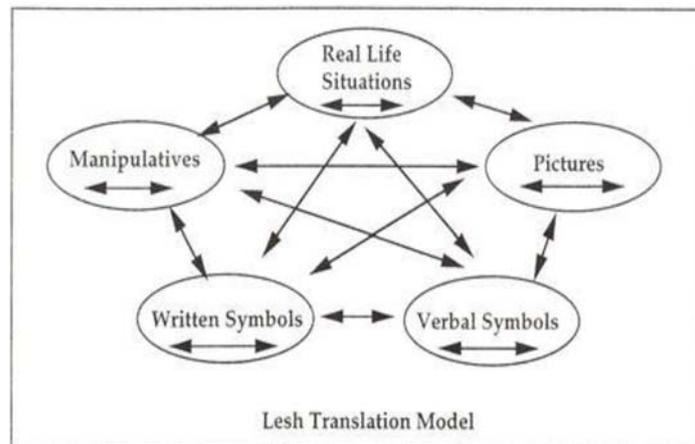


Figure 2.2 Lesh Translation Model (Lesh et al., 2003)

This model is built on the theories of Piaget, Bruner and Dienes. It suggests that basic mathematical ideas can be expressed in five different modes: manipulative, pictures, real-life contexts, verbal symbols and written symbols. This model stresses that understanding is reflected in the ability of the learners to represent mathematical ideas in multiple ways and the ability to make connections among the different embodiments. This model emphasizes that translation within and between various modes of representation makes ideas meaningful to learners.

This model suggests that the development of deep understanding of fractions requires experience in different modes, and experience in making connections between and within these modes of representation. A translation requires a re-interpretation of an idea from one mode of representation to another. This movement and its associated intellectual activity in the learners reflect a dynamic view of instruction and learning.

The Lesh model which emphasises flexible thinking, fits well with the adaptive reasoning strand of mathematical proficiency which emphasises reasoning about mathematics. In addition, the ability to move between the representations in the Lesh model would indicate good understanding and hence the Lesh model is closely associated with the conceptual understanding strand. Consequently, the notion of mathematical proficiency and

the Lesh translation model support and complement each other in providing a framework for this study.

The Lesh Translation Model (Lesh, et al., 2003) shown in Figure 2.2 was used in developing the RNP activities that were used for this study

2.4 THE RATIONAL NUMBER PROJECT (RNP)

“What the child can do in co-operation today, he can do alone tomorrow” (Vygotsky). Learners must be assisted today through the use of selected concrete teaching and learning materials so that in future they will solve problems alone without the teacher’s assistance.

Fraction lessons for the middle grades comprised two volumes of carefully researched lesson plans aimed at developing learners’ number sense of fractions (Cramer et al., 1997a, 1997b). Level 1 was written for learners in grades 4-5 and level 2 is written for grades 5-8. The model was based upon several key ideas. As a basis and to develop fraction number sense, learners spend time investigating concepts of order, equivalence, unit, addition and subtraction with concrete teaching and learning materials such as , fraction circles, counters, number line and paper folding. Another aspect of the programme was that each of the models used are analysed to see how they are alike or different and efforts are made to connect ideas across numerous types of representations. This practice is similar to the Lesh Translation Model.

Level 1 and 2 lessons emphasized developing the meaning of fraction symbols before asking learners to operate on them. Both sets of lessons covered the same topics, except for the fraction multiplication topic, and the introduction of a new, more complex model for fractions in level 2 lessons.

Initial lessons in level 1 had learners engage in modelling and naming (verbally and symbolically) fractions less than 1 using area models such as fraction circles, paper strips and other shapes. Throughout these initial lessons the concept of the flexibility of the unit was developed by using a variety of non-standard shapes such as half of a circle. Fraction equivalence and ordering were then introduced using area models before learners were asked

to develop the same concepts using a discrete model such as counters. In my research I substituted beans for the counters.

The next tier of lessons returns to the initial area models of fractions to develop learners' ability to reconstruct the whole, given a fractional part, and to model and name fractions greater than 1. A subsequent lesson extended the concept of fraction equivalence using the rate series by having learners look for number patterns in the information they have already gathered about equivalent fractions.

In the closing lessons of level 2, from which the research lessons were taken, addition, subtraction and multiplication are introduced by learners' modelling of stories. This is the special emphasis given to learners' estimation of the sums, differences or products by recognizing the approximate size of the fraction operands using their internalized visual models of fractions.

In order to assess and teach for conceptual understanding the RNP team created activities, task-based assessment and interviews that probed for conceptual understanding rather than procedural fluency. The RNP activities included paper folding and number lines and the assessment tasks included area models with perceptual distracters. An example of this is where a learner is asked to show a third of a rectangle which has been pre-drawn (Lesh et al., 1989).

RNP highlighted the importance of learners' abilities to represent mathematical ideas in multiple modes including manipulative, real life situations, pictures, verbal as well as written symbols. The RNP activities helped to dispel incorrect perceptions that school mathematics is unrelated to a learner's everyday experiences. They were designed to develop the language necessary for meaningful communication in mathematics and problem solving skills, confidence and enjoyment of mathematics.

2.5 THE PLACE OF FRACTIONS IN THE MATHEMATICS CURRICULUM

In order to contextualise the study, the place of fractions in the mathematical curriculum in South Africa is discussed in this section. To provide an international perspective, the curricula of Australia and Canada are also briefly discussed.

The South African Mathematics Curriculum. This research was conducted in 2009 when the Revised National Curriculum Statement (RNCS), (Department of Education (DoE),

2002), had been introduced. This policy emphasised the need for a shift from the traditional approach to outcome based education. It was underpinned by constructivism and inquiry based learning. The RNCS was introduced in an attempt to bring about fundamental changes to the Mathematics Curriculum, how it is taught and learned (DoE, 2002). The curriculum encouraged a learner-centred and activity based approach to education. Problem solving strategies to be used included: looking for patterns, drawing pictures, using tables and diagrams, estimating and predicting. The RNCS states that “being mathematically literate enables persons to contribute to and participate with confidence in society”. The mathematics learning area statement of this document follows the above statement that “access to mathematics is therefore a human right in itself” (DoE, 2002, p. 4).

Mathematics for Grades R-9 has five learning outcomes namely: Numbers, Operations and Relationships; Patterns, Functions and Algebra; Space and Shape; Measurement; and Data Handling. For this study I concentrated on the Learning Outcome One: Numbers, Operations and Relationships. This outcome is described as follows: “the learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems” (DoE, 2007, p. 70).

The Grade 7 Assessment Standards are reproduced below:

We know this [the outcomes have been achieved] when the learner:

- Describes and illustrates the historical and cultural development of numbers (e.g. Integers, common fractions).
- Recognises, classifies and represents the following numbers in order to describe and compare them: Integers: Decimals (to at least three decimal places), fractions and percentages: Factors including prime factors of 3 – digit whole numbers:
- Recognises and uses equivalent forms of the rational numbers listed above, including: Common fractions, Decimals, Percentages.
- Solves problems that involve ratio and rate.
- Estimates and calculates by selecting and using operations appropriate to solving problems that involve: addition, subtraction and multiplication of common fractions: division of positive decimals with at least 3 decimal places by whole numbers: finding percentages: exponents.
- Recognises, describes and uses: algorithms for finding equivalent fractions: the commutative, associative and distributive properties with positive rational numbers

and zero (the expectation is that learners should be able to use these properties and not necessarily to know the names of the properties). (DoE, 2007, p. 70).

In the fraction section the learner in Grade 7 should continue to build their understanding in illustrating, comparing and ordering fractions, such as to use half and one as benchmarks. The teacher can ask learners to state which benchmark a fraction is closer to and name a fraction that is even closer to the benchmark or give learners varied opportunities to build understanding of the addition, subtraction, multiplication and division of fractions using concrete teaching and learning materials and drawings before moving to symbols.

The Australian Mathematics Curriculum. The Australian Mathematics Curriculum begins with the simple representation of fractions and moves to more complex representation. The teacher first introduces the fraction concept using concrete teaching and learning materials, and then continues the concept with or without the use of concrete teaching and learning materials. The concept of fractions is taught from concrete to abstract following the constructivism theory. The details of Grade 7 work are reproduced below.

- By the end of Grade 7, students will:
- Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on the number line.
- Exploring equivalence among families of fractions by using a fraction wall or a number line (e.g. by using a fraction wall to show $\frac{2}{3}$ is the same as $\frac{4}{6}$ and $\frac{6}{9}$).
- Exploring and developing efficient strategies to solve additive problems involving fractions (e.g. by using fraction walls or rectangular arrays with dimensions equal to the denominators). Multiply and divide fractions and decimals using efficient written strategies and digital technologies.
- Investigating multiplication of fractions and decimals, using strategies including patterning and multiplication as repeated addition, with both concrete teaching and learning materials and digital technologies, and identifying the processes for division as the inverse of multiplication.
- Express one quantity as a fraction of another, with and without the use of digital technologies. Using authentic examples for the quantities to be expressed and understanding the reasons for the calculations.

- Round decimals to a specific number of decimal places. Using rounding to estimate the results of calculations with whole numbers and decimals, and understanding the conventions for rounding.
- Connecting fractions, decimals and percentages and carry out simple conversions. Justifying choices of written, mental or calculator strategies for solving specific problems including those involving large numbers.
Understanding that quantities can be represented by different number types and calculated using various operations, and that choice need to be made about each.
Calculating the percentage of the total local municipal area set aside for parkland, manufacturing, retail and residential dwellings to compare land use.
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. Using authentic problems to express quantities as percentages of other amounts.
- Recognise and solve problems involving simple ratios. Understanding that rate and ratio problems can be solved using fractions or percentages and choosing the most efficient form to solve a particular problem.

(The Australian Curriculum Assessment and Reporting Authority, 2013, p. 17).

The Canadian Mathematics Curriculum. By the end of Grade 7 the learner according to the Canadian Mathematics Curriculum will be able to represent, compare and order numbers including integers, and demonstrate an understanding of addition and subtraction of fractions and integers and apply a variety of computational strategies to solve problems involving whole numbers and decimal numbers. The learner must also demonstrate the understanding of proportional relationships using per cent, ratio and rate. The details of the curriculum are reproduced below.

By the end of Grade 7, students will:

- Represent, compare and order decimals to hundredths and fractions, using a variety of tools (e.g. number lines, base ten materials, calculators).
- Select and justify the most appropriate representation of a quantity (i.e. fraction, decimal, per cent) for a given context (e.g. “I would use a decimal for recording the length or mass of an object, and a fraction for part of an hour”).

(Ministry of Education, 2005, pp. 99-100).

In Canada and South Africa Grade 7 is seen as the last year of the middle (primary) school, but in Australia grade 7 is the first year of high school education.

The South African, Australian and Canadian curricula for Mathematics all advocate that the concept of fractions should be taught using concrete teaching and learning materials. According to Kilpatrick et al. (2001), concrete teaching and learning materials allow learners to correctly explore and investigate mathematical relationships that will later be translated into a symbolic form. They serve as alternative representations of concepts and should be used to introduce them. The key to the successful use of the concrete teaching and learning materials lies in the bridge (which must be built by the teacher) between the artefact and the formalized statement of the underlying mathematics concepts such as fractions. A weak link can defeat the purpose of the use of concrete teaching and learning materials (Kilpatrick et al.,2001).

The curricula also emphasize the learner centred instruction because they provide time for learners to reflect and gain a deep understanding of fractions. If a learner struggles to solve problems, the role of the teacher becomes one of active listening, clarification of issues and probing learner thinking. Classroom assessment is a tool that can help the learners to monitor their own learning. Learners need the opportunity to show both their ability to perform mathematics skills and their ability to apply them, and that is mathematical proficiency.

All the curriculum documents mention numerous strategies that could be implemented such as worksheets, hands-on and minds-on learning activities, learner-centred instruction, on-going assessment, problem-solving, small-group work and individual work.

2.6 ISSUES IN LEARNING FRACTIONS

Piaget (1960) found that the difficulties that learners have in understanding fractions could be explained as the result of a conflict between new information and their prior knowledge. He explained that he firstly introduced learners to the idea of number (cardinal and ordinal aspects), symbolism and the operations before introducing fractions. He said that when children were introduced to fractions there might be no suggestion that fractions are themselves also numbers, but he hoped that what is regarded by the learner as a number would be extended and modified from the original implication that 'numbers' imply

natural number, including zero. In assimilating the ideas that fractions are rational numbers, that improper fractions are still fractions the previously held view of what number means requires modification. Piaget (1960) said assimilation cannot take place without accommodation, and accommodation might not be easy. For example, when asked to choose a number almost everybody assumes whole number is required.

In the specific area of fractions, researchers Chi, Slotta and de Leeuw (1994) have argued that fractions represent a case of a concept, the acquisition of which requires name change. However, Lesh et al. (1992) showed that one of the reasons why the mathematical notion of a fraction is systematically misrepresented is because it is not consistent with the counting principles that apply to natural numbers. They concluded that early knowledge about number may in fact serve as a barrier to learning about fractions, given learners' constructivist tendency to distort the new information about fractions to fit their counting based number theory.

Lesh et al. (1992) point out that fractions differ from natural numbers in significant ways. Firstly in their representation: one number versus two cardinal numbers separated by a line. Secondly with respect to ordering: fractions differ from natural numbers in that one cannot use counting based algorithms for ordering them. Fractions do not have successors: there are infinitely many numbers between any two fractions. Thirdly with respect to the unit: while the unit is the smallest natural number, there is no 'smallest' fraction. Finally four basic operations such as addition, subtraction, multiplication and division on natural numbers differ in important ways from operations performed in fractions.

The concept of a fraction is difficult because it includes addition, subtraction, multiplication, division and simplification: that is there are various operations that learners do with fractions. The amount of rules that are there to be learnt about working with fractions is problematic. These are presented in standard textbooks as shown in Table 2.1.

Table 2.1 Fraction operations and rules

OPERATION	RULE
1. Fraction addition, with same denominator.	Add the numerators, use the same denominator.
2. Fraction addition, with different denominator.	Find the LCD for the fractions being added, and then add the numerators.
3. Fraction multiplication.	Multiply both numerators and denominators. (Do not need LCD).
4. Fraction division.	Invert (turn upside down) the second fraction, then multiply.

(Source: Just Mathematics 5, 1990, p. 70)

These are the four rules that learners need to know before a teacher can be sure that they have understood lessons on fractions and that they can do the required computations.

Human, Olivier, Le Roux and Murray (2002), talk about the background that is needed in the teaching and learning of the concept of a fraction, such as common and equivalent fractions. If this foundation knowledge is not in place, then introducing decimal fractions causes confusion and may break down whichever concepts the learners may have had of common fractions. Necessary pre-knowledge to the understanding of common fractions includes common fraction notation, an understanding of whole numbers up to and beyond a thousand, which implies a feeling for sizes of the larger numbers as well as a good understanding of denseness (closely packed) of fractions. Human et al. (2002) stress the importance of using concrete teaching and learning materials such as measuring scales and number lines. The learners need to read scales and number lines and interpret their readings. The readings have an effect when estimating and multiplying and dividing with decimal and equivalent fractions.

2.7 CONCRETE TEACHING AND LEARNING MATERIALS IN MATHEMATICS

Concrete teaching and learning materials are objects that may be viewed and physically handled by learners in order to demonstrate or model abstract mathematical concepts. In this section, the importance of bridging the divide between concrete experience and abstract mathematical concepts is discussed, followed by a discussion of the literature motivating the use of concrete materials. Finally the specific concrete materials used for developing fraction concepts in this study are described.

Bridging from the concrete to the abstract. Reys et al. (1992) metaphorically suggest ten planks making up a bridge that serves to connect concrete experiences with mathematical concepts. The ten planks for the bridges are: mathematics learning should be meaningful, mathematics learning is a developmental process, mathematics learning should build on previous learning, learners should be actively involved in learning mathematics, learners need to know what is to learned in mathematics classrooms, communication is an integral part of mathematics learning, multi embodiment aids learning mathematics, mathematical variability

aids learning, metacognition affects mathematics learning and forgetting is a natural aspect of learning, but retention can be aided. Of particular relevance to this study is the idea of multi embodiment which relates to many representations of the fraction concepts. Children need to experience many different embodiments so that they can recognise the attributes shared by all embodiments, and hence generalise (Reys, Lindquist, Lambdin & Smith, 2012). This view is supported by Pantziara and Philippou (2012) who report that their study suggested that the use of representations, and the movement between the representations are important for the development of the learners' conceptual knowledge.

Bridges are effective at all stages of teaching and learning such as introduction, review, problem solving and assessment. Reys et al. (1992) say there are three purposes for the bridges. Firstly, bridges link real world applications with textbook mathematics. This bridging not only clarifies concepts, but increases learners motivation. Secondly, bridges provide a connection between instructional models and formal mathematical ideas and symbols. Thirdly, bridges provide a path that can be travelled many times, in either direction to reach greater understanding. Frequent use of bridges serves to narrow the gap between the concrete and abstract, thereby promoting meaningful learning and greater retention.

2.8 MOTIVATION FOR THE USE OF CONCRETE TEACHING AND LEARNING MATERIALS.

“Fractions are better understood when seen” (Anonymous).

The above quote reminds us that the concept of fractions is difficult to teach and learn, and teachers need to use concrete teaching and learning materials to assist the learners to learn this concept with understanding. Fractions differ from whole numbers in many ways, the most important being that the “whole” varies from one context to another, and may be continuous or discrete (Siemon et al., 2013).

The ideas of Piaget (1960), Dienes (1960) and Bruner (1961) continue to inspire teachers to use concrete teaching and learning materials in the teaching and learning of mathematics. Concrete teaching and learning materials provide the learners with a tangible resource to use in learning mathematics. They assist learners to draw on their practical, real-world knowledge. They also induce physical action to enhance memory and understanding. (Reys et al., 1992) compare the theoretical framework suggested by theorists such as Piaget, Dienes and Bruner in the teaching and learning of mathematics (see Table 2.2). The framework has different operational stages that should be followed, beginning from concrete

to more abstract stages. This provides motivation for the use of concrete teaching and learning materials.

Table 2.2 Comparison of Frameworks of the learning Process:

Piaget Stages	Bruner Modes	Dienes Model
Pre operational Represents action through thought and language.	Enactive Learner is interacting directly with physical world.	Free Play Learner interacts directly with concrete teaching and learning materials.
Concrete Operational Thinking is perceptual and limited to physical reality.	Iconic Learner is involved with pictorial/verbal information based on real world.	Generalization Patterns, regularities and commonalities are observed. Representation Provides a peg on which to hang what has been abstracted.
Formal Operational Capable of logical thinking, reflect on their own thought processes.	Symbolic Learner manipulates symbols irrespective of their enactive/iconic counterparts.	Symbolization Describes the representation in language/mathematical symbols. Formalization Fundamental rules and properties are recognized as a structure of mathematics.

(Source: Reys, Suydam & Lindquist, 1992, p. 49)

Kanduz and Rudolf (2004) emphasize the importance of the learners creating mental images during mathematics lessons, because learners will see the situation in more than one way and will have numerous responses to one question. They mention two kinds of images, namely visual material images (pictures) and verbal images (metaphors), which are linked by similitude (similar). Their main argument is that learning mathematics can be described as a continuous interplay of images and diagrams which are linked by metaphors. A teacher can use a number line when teaching the concept of a fraction, so that in the learners' minds the number line will interplay with fractions.

Skemp (1976) identified two types of mathematical understandings which are associated with patterns of thinking, namely instrumental and relational understanding. He suggested that conceptual and procedural understandings appear to be dominant amongst the

instrumental thinkers. For example, pupils learn to divide fractions by inverting and multiplying as a recipe without any understanding. It is then procedural and the beauty of doing mathematics is hidden.

In contrast to instrumental understanding, strategic competence, adaptive reasoning and productive disposition are dominant amongst the relational thinkers. They appear to reject, temporarily ignore or select information which is more relevant to the task. Relational thinkers give direction to learners rather than pump in knowledge and skills. They believe that concrete teaching and learning materials emphasize hands-on learning, where the learners use the materials to construct their own knowledge of mathematics (Skemp, 1976).

Thornton (2001) argues that the use of concrete teaching and learning materials such as number lines in the teaching and learning of mathematics is of crucial importance, because they promote flexibility in thinking and enable learners to move from formal to informal and vice versa. Learners need to be constantly exposed to concrete teaching and learning materials in order to stimulate their level of cognition, such as to locate $\frac{1}{2}, \frac{1}{4}, \frac{5}{6}, \frac{9}{9}$ on the number line. He claims that a picture is worth a 'thousand words' and that visualization provides a simple, elegant and powerful approaches to develop mathematics results by making connections between different areas of mathematics such as algebra. Visualization serves to deepen learners' understanding of mathematics concepts such as fractions (Thornton, 2001).

Whitely (2004), a research mathematician, says his practice of mathematics is deeply visual (i.e. he uses concrete teaching and learning materials). He argues that the modern abstract and applied mathematics can be intensely visual by combining a high level of reasoning with solid grounding in the senses. Concrete teaching and learning materials are used in diverse ways by practicing mathematicians. By contrast what learners see in a mathematics classroom is very abstract. He made nine claims about concrete teaching and learning materials which are: visuals are widely used in diverse ways by practicing mathematicians; visual reasoning in problem solving is central to numerous other fields such as engineering, computer science, chemistry, biology and applied statistics; children use visual processes for early work in mathematics; visual reasoning is not restricted to geometry or spatial represented mathematics; we create what we see 'seeing to think'; visual and diagrammatic reasoning is cognitively distinct from verbal reasoning; visual reasoning is connected to kinaesthetic and emotional reasoning; children begin school with relevant visual

abilities, including 3-D; and visually based pedagogy opens mathematics to learners who are otherwise excluded (Whitely, 2004).

The view of the importance of concrete materials is supported by Kotagiri (2002), who used concrete teaching and learning materials when he was researching learners with difficulties in learning mathematics. He empirically demonstrated the possibility of learners overcoming learning difficulties through the use of concrete teaching and learning materials. He assisted the learners by going through four steps namely: real world where tangible objects are manipulated, model world uses pictures, schema where idealized models are being defined and lastly the mathematical world where thinking is expanded by manipulating numbers and characters. He further said that concrete teaching and learning materials such as computers encourage learners to rehearse the mathematical operations already acquired, make sense of the significance of their learning and encourage them to discover new mathematical ideas and operations. He said that by observing the learners' activities, teachers can see the learners' thinking and evaluate how each learner's learning is progressing (Kotagiri, 2002).

Despite all the literature supporting the use of concrete materials, some scholars caution against the way in which they are used. Smith (1996) and Moyer (2001) recommend that concrete teaching and learning materials should not be used solely for fun or to add variety to mathematics lessons, but should be used to engage learners in mathematics. Concrete teaching and learning materials must not be used to arouse learners' interest, but to teach them mathematics concepts and procedures. They also suggest that teachers consider concrete teaching and learning materials to be fun, but fail to recognize their value as tools for teaching and learning mathematics.

Recent researchers such as Fennema (1972), Moyer (2001) and Amaya (2007) caution that concrete teaching and learning materials do not guarantee success, they begin to lose their usefulness after Grade 1. They have no or little impact on learners' understanding of mathematical concepts and ability to use knowledge to solve written exercises.

2.9 CONCRETE TEACHING AND LEARNING MATERIALS USED IN THE STUDY

*“I hear and I forget,
I see and I remember,
I do and I understand”.* (Confucius 551 – 479 B.C.)

The above proverb illustrates the importance of using concrete teaching and learning materials when teaching abstract concepts in mathematics such as fractions. The particular concrete materials used in this study are described in this section.

Cuisenaire rods

Cuisenaire rods are bars that use length and colour to represent fractions. They were used to demonstrate equivalent fractions, shown by trains (rods line up end to end) of the same length. There may be several groups of equivalent fractions for each unit. The numerator of each fraction is the number of rods used in the fraction. The denominator of each fraction is the number of rods that are used if the train is equal to the unit.

Colour	Length (cm)
White	1 cm
Red	2 cm
Light Green	3 cm
Lavender	4 cm
Yellow	5 cm
Dark Green	6 cm
Black	7 cm
Brown	8 cm
Blue	9 cm
Orange	10 cm

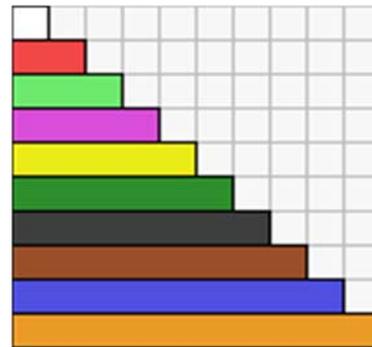


Figure 2.3 Diagram showing the full set of Cuisenaire rods.

Cuisenaire rods are concrete, physical objects which a learner can see, touch and manipulate in the classroom (Kurumeh & Archer, 2008). I used the rods approach to teach fraction concepts because they brought about improved performance in the learners in the fraction activities as well as in the fraction assessment. Rods were used as both the concrete

teaching and learning materials as and as the symbolic concrete representation in the teaching and learning of fractions.

Cuisenaire rods provide a hands- on and minds-on, activity-filled approach to learning fractions. Because the rods are ready made tools, this approach minimizes preparation and set up time both for the teacher and learner. This approach develops skills such as classification, critical thinking, problem solving and mathematical and spatial reasoning. Their use is crucial in developing learners’ understanding of fraction ideas. They help learners construct mental referents that enable them perform fraction tasks meaningfully. The use of concrete teaching and learning material for practical takes away the abstractness seen in mathematics concepts (Kurumeh & Acher, 2008).

Fraction circles

The fraction circles set used in this study consisted of six circles that show wholes, halves, thirds, fourths, fifths and sixths. The teacher’s set had magnetic strips on the back so that the pieces could be stuck to the chalkboard. Fraction circles are normally used to engage learners in exploring and discovering relationships between the pieces, and sometimes to solve problems that require the four basic operations such as addition, subtraction, multiplication and division of fractions. Fraction circles may also be used to record fractions that name the different pieces or combination of pieces.

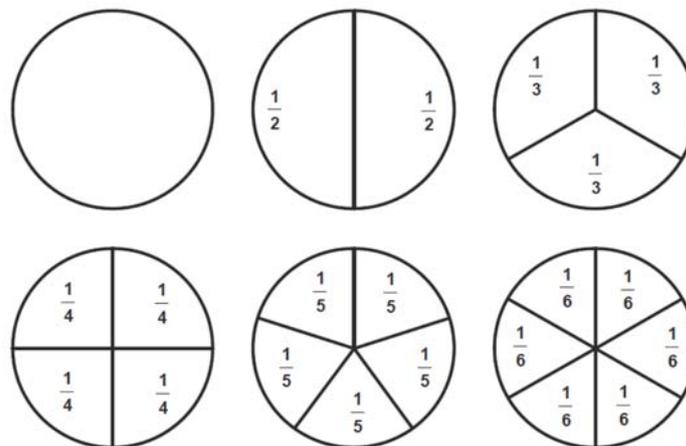


Figure 2.4 Diagram of fraction circles

Fraction Strips:

In this study I used 20cm paper strips. The strips can be folded and learners asked for example, how many parts are there? How many folds are there? What do we call each part? Show me one-half of the paper strips? Fraction strips provide a concrete foundation for

comparison activities. Learners can compare the relative sizes of unit fractions given different wholes, such as ‘if the dark green strip is a whole, which strip is one-half, one-fourth, or one-fifth?’

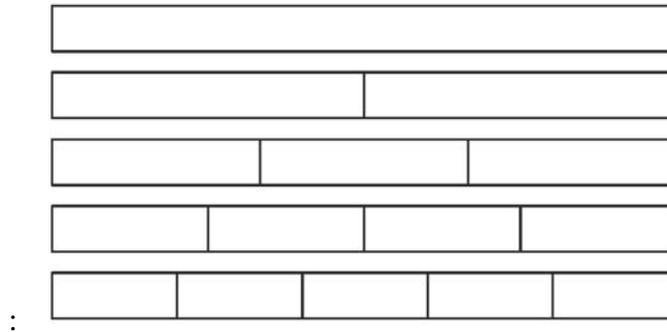


Figure 2.5 Diagram showing fraction strips

Discrete materials:

The understanding of fractions as a part of a set of discrete objects or counters (as opposed to a continuous whole) requires a large number of discrete objects. Different varieties of beans are useful in this regard as they are cheap and readily available. For this study, white and brown beans were used as concrete teaching and learning materials.

Number lines

Number lines can be used to assist learners to develop a visual understanding of the numerical value of fractions and to develop an understanding of fractions as parts of a whole. Number lines assist learners to know how to calculate unit fractions of a range of numbers. First, learners should count how many sections the number line is broken into (they should count from one whole number to another). This number becomes the denominator of the fraction. The learners should count to see how far an arrow is placed on the same line. This number becomes the numerator of the fraction. Below is the illustration (Figure 2.6), where the task is to determine the fraction represented by the arrow on the number line.



Figure 2.6 Example of a number line used to develop fraction concepts

2.10 LANGUAGE ISSUES IN LEARNING MATHEMATICS IN SOUTH AFRICA

The language of teaching and learning (LoTL) in South African primary schools is decided by the governing body of individual schools. (DBE, 2010) At my school the SGB has chosen English language as the LoLT. The language policy (DBE, 2010) maintains that learners acquire knowledge better when they learn in their mother tongue, especially in the formative years (foundation and intermediate phases). It also emphasises that no learner should be left behind in mathematics because of the LoLT chosen by the governing body of the school. The teacher must link second language learning and the learning of mathematics through code-switching, to promote easy communication in order to improve learner's mathematical reasoning skills through social interaction. The crucial issue is that the teacher must make informed choices so that the dialogue supports the understanding of both the language and the mathematical concepts (fractions) for all learners in the classroom. Learners think in a language and they use the language to obtain help from others and solve mathematics problems (DBE, 2010).

In the RNCS Grades R-9 (DoE, 2003, p. 13) it is noted that “teachers should understand that learning in a language which is not one's home language needs to be addressed through the way they teach, plan activities and assess learner performance”. The strategy of developing exploratory talk among learners through dialogue in the language they understand the best is a way to overcome the language barrier to learning mathematics. Setati and Adler (2001) conducted research into multilingual and multicultural mathematics teaching and learning in South Africa. They described code switching as alternative in use of more than one language in a speech act or code switching is a communication model that is used in mathematics classrooms where the LoLT is English. A teacher can switch codes in order to translate or clarify instructions, but also to reformulate and model appropriate mathematical language use (Setati and Adler, 2001). They noted some dilemmas which are challenges for all teachers who use code switching in multilingual classrooms where informal spoken mathematics is not the LoLT. In a classroom where learners home languages are different, such as Xitsonga, Sepedi, isiXhosa, Setswana and isiZulu, a teacher must first get a common word in Nguni language (iqhezu to break) that will accommodate all the languages

spoken in the classroom before using an English word for a mathematical concept. In such classrooms, learners are acquiring English while learning Mathematics.

In order to link second language learning and learning of mathematics Setati and Adler (2001) promote the use of dialogue in the learner and teacher's main language such as vernacular. Here learners are free to discuss mathematical concepts in their home language. This is then used together with code-switching, to promote easy communication in order to improve learners' mathematical reasoning skills through social interaction.

The main aim of code-switching is to ensure that learners have the opportunity to experience and discuss concepts in the language they are most familiar with and then learn to explain and use them in English. Teachers may feel guilty when they code-switch thinking that they are depriving learners of an opportunity of acquiring the English language (Setati & Adler, 2001) but through code switching technique learners are learning mathematics concepts with understanding and thereby avoiding verbalism (knowing a concept without understanding its meaning).

Setati and Adler (2001) caution that learners may sometimes find the syntax of mathematical discourse difficult such as word order, logical structures and conditionals are all particularly problematic for them. Learners may also be unfamiliar with the contexts in which problems have been situated. Code switching which involves the teacher substituting a home language word for a mathematical word has been shown to enhance learner's understanding, especially when teachers are able to use it to capture the specific nuances of mathematical language.

2.11 SIMILAR STUDIES

The effectiveness of concrete teaching and learning materials in mathematics education has been the subject of extensive research. Suydam and Higgins (1977) performed a meta-analysis of 40 research studies into the use and effectiveness of concrete teaching and learning materials on the learner achievement in mathematics. 60% of the studies indicated that concrete teaching and learning materials had a positive effect on learner learning, 30% showed no effect on achievement, and 10% showed significant differences favouring the use of more traditional (non-concrete teaching and learning materials) instructional approaches. In similar work, Sowell (1989) performed a meta-analysis of 60 additional research studies into the effectiveness of various types of concrete teaching and learning materials with

kindergarten through post-secondary learners. On the basis of the research, she concluded that achievement in mathematics could be increased through the long-term use of concrete teaching and learning materials. Wearne and Hilbert (1991) reported consistent success in the use of concrete teaching and learning materials to aid students' understanding of decimal fractions and decimal numeration.

The value of teaching using examples from the real world received attention from Streefland (1990) and Ball (1993). Streefland (1990) defended and provided examples of approaches to the teaching-learning of fractions within the real world to justify step by step the needs that derive from daily life as regards the learning and mastering of fractions and rational numbers. One of Streefland's main objectives was to stimulate learners to develop a '*fraction language*' as a way of understanding the concept of fractions. For example, he combined activities of fair sharing with classroom discussions, thus binding natural language related to the sharing process (each child will get one whole and three – fifths) to formal fraction language (each will get 1 whole and $\frac{3}{5}$). His research emphasized that concrete teaching and learning materials were used to bring the real world into the mathematics classroom. Ball (1993), presented a personal teaching – learning experience with learners in the third year of primary school, involving long discussion of everyday uses of fractions in ordinary language, the construction of a solid awareness and the consolidation of personal representations before she discussed the use of symbols, which are openly negotiated.

The issue of the causes of the difficulties learners had in the necessary transition from concrete experiences to formal reasoning, and in the representation model of fractions, as well as in the many subsidiary concepts needed to obtain fraction proficiency were addressed extensively by Behr, Lesh, Post, Silver (1993) of the RNP. They provided a critical discussion of teaching activities that are contemporary and distinguish between stages in the learning of fractions and rational numbers and analysing the language of fractions didactics in the classroom.

A similar study was carried out in United States of America (Naiser, Wright, Capraro, 2004) where the researchers focused not on the content being taught, but to identify effective pedagogical strategies used by middle school teachers in order to find ways to improve fraction instruction. The data was collected for four months in twelve classrooms. One of the areas of improvement is the use of concrete teaching and learning materials to make lessons more engaging by creating hands-on experiences. This study shows that by using concrete teaching and learning materials learners improved in their understanding of the concept of fraction.

Another study was carried out in Nigeria by Kurumeh and Achor (2008), in which the researchers wanted to find out the effect of using teaching and learning materials on learners' achievement in fractions. They found that the use of the teaching and learning materials to the teaching of fractions is more facilitative in enhancing learners' achievement than the conventional method of teaching.

The above studies acknowledge the importance of providing the learners with real life experiences provided by concrete teaching and learning materials. There are however some cautions, particularly around the age of the learners involved. Fennema (1972) argued for the use of concrete teaching and learning materials with beginning learners, while maintaining that older learners would not necessarily benefit from them. His research cautions teachers about the age of the learners for whom concrete teaching and learning materials in the mathematics classrooms would be beneficial.

For learners to transfer and apply knowledge to new situations requires learning that is founded in real life situation. Hands-on learning allows learners to build a functional understanding that can be applied in unfamiliar situations. When using concrete teaching and learning materials teachers act as facilitators. They help learners to discover and focus on the mathematical concepts and help them build bridges from concrete work to corresponding abstract work (Fennema, 1972).

In my opinion and based on the literature review, there is no single best way to teach mathematics however, the above studies show that using concrete teaching and learning materials in conjunction with other strategies can deepen learners' understanding of abstract concepts such as fractions. Appropriate use of concrete teaching and learning materials could be one strategy for teaching mathematics in the classroom.

2.12 SUMMARY

This chapter provided a literature review for this study. This chapter covered issues such as learning of mathematics in general, learning of fraction concepts specifically for this study using RNP activities, the Lesh translation model, Mathematical proficiency and the importance of concrete teaching and learning materials in developing mathematics proficiency. The issue of language as a barrier to learning mathematics was also discussed. Chapter Three describes the research methodology.

CHAPTER 3 RESEARCH METHODOLOGY

In this chapter I describe and motivate the research activities I carried out during the course of the study. Qualitative data was collected largely through using RNP activities and this was subjected to inductive analysis. Three cycles of action research were conducted within an interpretive research paradigm. Quantitative data were derived from tests which followed the teaching intervention and the end of year examination marks. These were used, respectively, to measure the achievements of the interventions and to validate the assertion that the enhanced intervention of the action research improved the learners' mathematical proficiency with regard to fractions.

The purpose of the study was to determine how Grade7 learners' proficiency in fractions is affected by the continuous use of a range of concrete teaching and learning materials. The research questions are repeated here for clarity:

1. Can selected concrete teaching and learning materials help in developing proficiency in fractions in Grade 7 learners?
2. Which pedagogical practices and strategies are helpful in developing mathematical proficiency in fractions?

3.1 RESEARCH APPROACH

The research approach was a pragmatic one, using action research to improve my practice and drawing on qualitative data to assess the impact and validity of the interventions I developed. That is to say it uses a mixed method approach, with qualitative and quantitative data to answer the research questions. (Johnson & Onwuegbuzie,2004).

Action research aims to contribute both to the practical concerns of the people in an immediate problematic situation and to the goals of social science by joint collaboration within a mutually acceptable ethical framework. (Kemmis & Mc Taggart,2000). That is to say it is carried out by practitioners, in this case myself, in their normal surroundings, in this case my school with my colleagues and learners, to solve a specified problem, in this case the remediation of teaching and learning fractions. Kemmis and Mc Taggart (2000) talk about four stages of an action research namely: study and plan, take action, collect and analyse evidence and then reflect. They describe this as a progressive problem solving action research cycle.

The use of action research to deepen and develop classroom practice has grown into a strong tradition of practice. Action research is a form of collective self-reflective enquiry undertaken by participants in social situations in order to improve the rationality and justice of their own social or educational practices, as well as their understanding of those practices and the situations in which the practices are carried out. The approach is only action research when it is collaborative, though it is important to realize that action research of the group is achieved through the critically examined action of individual group members (Kemmis and McTaggart, 1988, pp. 5-6).

Figure 3.1 illustrates the cyclical nature of action research. The experience on Cycle 2 should be an improvement on Cycle 1 and Cycle 3 should be an improvement on Cycle 2. The names of the classes that were used in this study have been added.

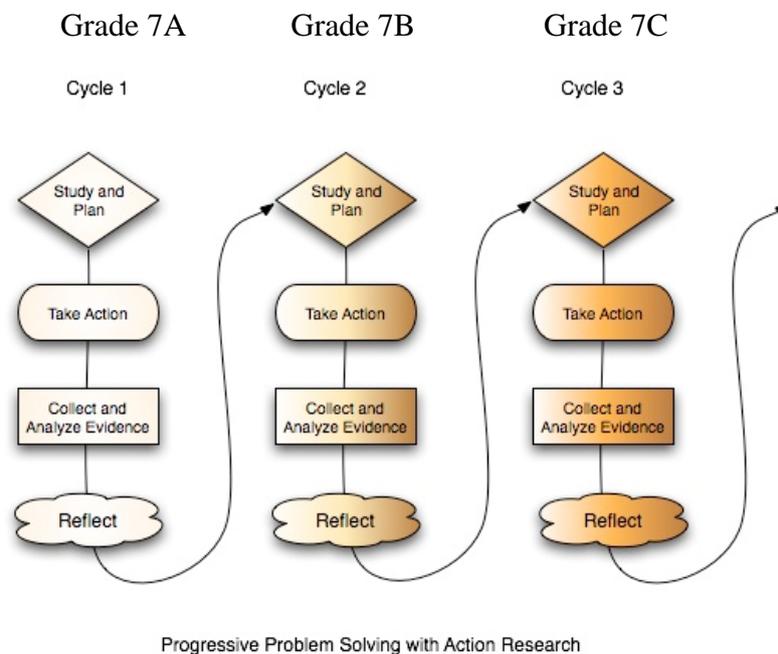


Figure 3.1 Diagram showing action research cycles (Source, Kemmis & McTaggart, 2000, p. 136)

I identify my study as an action research project because I wanted to improve my professional practice through continual learning and progressive problem solving. I wanted to develop and deepen classroom practice in a way that would lead to increased mathematical proficiency in the learners. I did this study with the intention that the project will inform and change my practices in the future. The plan of this project involved me as researcher, in consultation with two critical friends investigating the use of concrete teaching and learning materials and strategies for teaching-learning fractions. I sought solutions to problems of

instructional strategies, use of manipulative and proficiency in mathematics (learners learning). I taught fractions in this study, because I believe that they are the gateway to study Algebra in the secondary school.

3.2 RESEARCH DESIGN

For this study I implemented action research, because I contributed in the formulation of the research questions and in the collection of data. The action research cycle that was followed for the study is: Study and plan, Take action, Collect and analyse evidence and Reflect. The researcher was linked with the sources of knowledge such as the school, critical friends (fellow teachers), Grade 7 learners, RNP activities and Fraction test.

Learners worked together and were divided into eight groups of six learners per class. Learning activities designed by the RNP to help the learners master concepts and remediate previous learning of fractions were implemented. I was a facilitator, assisting groups with difficulties toward the discovery of solutions. Guided discovery encouraged learners to be active in the learning environment. I incorporated various teaching strategies into each lesson, including direct instruction, guided discovery, whole class discussion, and co-operative learning.

The study was conducted in the primary school where I was a Grade 7 mathematics and natural science educator. The school is situated in Pinetown district in KwaZulu-Natal. The learners are all African from different ethnic groups such as Xhosas, Sothos and Zulus. I taught mathematics to all three heterogeneously mixed Grade 7 classes (grades 7A, 7B and 7C). The biographical characteristics of the learners in this study are presented to give some context to the study. There were eighty-two girls (55%) and sixty-eight boys (45%). Their ages ranged from 12 years to 17 years. The appropriate age for Grade 7 is 12 -14. There were few learners with ages 15, 16 and 17 years, may be they were retained or fell pregnant the previous year and left school.

The initial sample was all the Grade 7 learners (N= 150) at the primary school where I teach. This sample was chosen as I was able to teach mathematics to all three classes and so could repeat the activities in the action research cycle. I used a combination of purposive and stratified selection methods to select learners (N=12) for the focus group interviews. Purposive selection methods involve the researcher handpicking the participants to be included in the sample on the basis of typicality and for the specific qualities they bring to the

study (Lankshear and Knoebel, 2004) Such a view is shared by Burns (1998), who contends that purposive sampling enables the researcher to select participants on the basis of their experiences and the knowledge they have. The size of the sample was considered sufficient as no generalization on the entire population was sought.

The research study was conducted over six weeks in the second school term. This primary school had three Grade 7 classes each with a class teacher who taught the learners several subjects. I was the class teacher for Grade 7C, and the specialist Mathematics teacher for all three classes. This enabled me to teach the same content three times over, each time reflecting and learning from my experience with the previous class. I asked the other class teachers (Ben and Jabu as pseudonyms in this study) to assist me as critical friends. I planned ten lessons on fractions and selected fifteen suitable RNP activities to use in these lessons. The critical friend teachers were asked to comment on this selection of RNP activities. I was able to establish a good working relationship with the teachers acting as critical friends.

I began the project with Grade 7A, taught ten sixty-minute fraction lessons. I divided the class into eight groups of six learners, and learners worked together to complete fifteen selected RNP activities. My critical friend (Ben) observed one lesson with Grade 7A and gave me a feedback after the lesson. After each lesson, I marked their completed RNP activities. Ben and I looked at the marked RNP activities together and discussed the success or the failure of the lessons. After the completion of the ten fraction lessons and fifteen RNP activities, I gave Grade 7A a fraction test to be written individually without assistance and the test scripts were marked.

The second group was Grade 7B the teaching sequence began all over again with insights gained from the first cycle. I taught ten sixty-minute fraction lessons, gave fifteen RNP activities and a fraction test at the end. The other critical friend (Jabu) observed Grade 7B. The last group was Grade 7C, Jabu again observed lessons in Grade 7C. This group also wrote the fraction test.

As the research design progressed other elements were introduced, such as that the learners wrote a fraction test. The tests were analysed statistically and compared with their end of the year examination marks in Mathematics and English language. The examination marks were also used to validate the assertion that the experience of class 7C in the action research process contributed to their improved mathematical competency.

3.3 DATA COLLECTION

The collection of data included observation of learners' interaction with the concrete teaching and learning materials, peers and critical friends, learners completing RNP activities in groups, their explanations of how things worked (interviews) and a written fraction test.

The final Grade 7 examination results were also obtained.

The data corpus is summarised in the table below.

Table 3.1 Data Corpus:

Instrument	Participants	Data Collected	Data Analysis	Answering
Unstructured Class Observation	Critical Teacher N=2	Written reports		RQ 1
RNP Activities	Grade 7 learners N=150	Learner Work on 15 activities	Analyse work for competence in aspects of Lesh Model	RQ 1
Learner Interviews	Stratified sample of learners selected N=12	Written record of responses to performance task	Lesh Model	RQ 1
Research Diary	Researcher	Researcher's Notes		RQ 1
Fraction Test	Grade 7 learners N=150	Learners' work and marks	Compare marks form different classes Mathematics Proficiency	RQ 2
Mathematics Examination	Grade 7 learners N=150	Marks of Learners	Mathematics Proficiency	RQ 2
English Examination	Grade 7 learners N=150	Marks of Learners	Literacy Proficiency Compare with maths marks	RQ 2

The Teaching Sequence

Below is a summary of the ten lessons derived from the Rational Number Project (RNP), which were taught in three repeating cycles. This is followed by a more detailed description of each lesson.

Table 3.2 Summary of Teaching Sequence showing links to RNP Activities

Cycle 1	Focus	Class activity	RNP Activity
Lesson 1	Familiarisation with the Cuisenaire rods	Activity 1: Exploring with	Level 2/Lesson 1 Student Page A
Lesson 2	Modelling fractions with $\frac{1}{2}$. Given the unit, learners are asked to find the fraction name for the other rods	Activity 2: Use $\frac{1}{2}$ to problems Activity 3: Use $\frac{1}{2}$ to do these problems Activity 4: Use $\frac{1}{2}$ to find units	Level 2/Lesson 3 Student page A Student page B Student page C
Lesson 3	Exploration of fraction equivalence with $\frac{1}{2}$ and discrete objects (beans)	Activity 5: Equivalence and	Level 2/Lesson 7 Student page A
Lesson 4	Modelling fractions with fraction circles and using benchmark of one half to order fractions	Activity 6: Fraction estimation Activity 7: Fraction estimation	Level 2/Lesson 8 Student page A Student page B
Lesson 5	Using mental images to order fractions with same numerators and different denominators	Activity 8: Mental images to circle larger fractions,	Level 2/Lesson 9 Student page A
Lesson 6	Change improper fractions to mixed numbers using mental images of fraction circles or	Activity 9: Mixed and Improper Fractions	Level 2/Lesson 11 Student page A
Lesson 7	Review fraction equivalence using $\frac{1}{2}$ and beans as models		
Lesson 8	Using paper folding to develop the multiplication rule for generating equivalent fractions	Activity 10: Bingo games Activity 12/13: This puzzle will make you famous Activity 14: Crack the Code	Level 2/Lesson 14 Student page C Student page A1,A2 Student page B
Lesson 9	Using $\frac{1}{2}$ to compare fractions	Activity 11 Activity 14: Order and equivalence with rods	Level 2/Lesson 10 Student page B Level2/Lesson 15 Student page A
Lesson 10	Using beans to find a common denominator for two fractions	Activity 15	Level 2/Lesson 16 Student Page A
Lesson 11	Test	Test: 20 short questions selected and adapted from RNP activities and assessments	

RNP Activities:

Some of the activities used in the lessons are described here. I have selected the ones I considered to be the most useful based on my experiences of the learners' responses.

Activity One: Familiarisation with the Cuisenaire rods. This activity served to introduce fractions as different to whole numbers. Longer rods were used as units and shorter rods used as the fractions of longer rods. They were given small tasks such as the one below.

A. Arrange your rods to represent each of the following fractions using the dark green rod as the denominator. For example,



I explained to the learners that the numerator of $\frac{1}{6}$ is the number of white rods used in the fraction (A) and the denominator is the number of white rods that would be required to equal the unit length (green rod train).

Activity Two: Using Cuisenaire rods to solve problems. I gave learners to model fractions such as $\frac{3}{5}$ and $\frac{2}{3}$. When learners were using the rods, they would say the fraction's name. The follow up activity was when I asked them to draw the pictures of their models on the activity sheets. The activity sheets were collected for assessing the learners' progress and understanding of the lesson.

Activity Three: Using Cuisenaire rods as units. I asked the learners to use the fraction trains to write fractions, e.g. red is what fraction of the orange train? I asked learners to first model the fractions with the rods, and later asked them to draw the pictures of the models on the provided spaces. I also gave them an activity sheet to complete to practice naming fractions.

Cuisenaire rods were used to demonstrate equivalent fractions, shown by trains (rods line up end to end) of the same length. In comparing any set of trains showing equivalent fractions, the train with the smallest number of rods represents the fraction in its lowest terms. There may be several groups of equivalent fractions for each unit. The numerator of each fraction is the number of rods used in the fraction. The denominator of each fraction is the number of rods that are used if the train is equal to the unit.

Activity Four: Using Cuisenaire rods to find units. I asked the learners to use the blue rods as a train to find the values of other rods such as white is $\frac{1}{9}$, dark green is $\frac{1}{2}$, and orange and brown are one whole each.

Activity Five: Equivalence and Cuisenaire rods. I asked the learners to use two different pairs of rods like orange and red train as their unit or blue rod as a unit, to model fractions: $\frac{4}{6}$, $\frac{1}{4}$ or $\frac{2}{3}$. They were to give another name for $\frac{1}{4}$ like $\frac{2}{8}$ or for $\frac{2}{3}$ like $\frac{4}{6}$. The next time in cycle two this activity was further done using brown as train to model other rods such as 1 white, 2 reds, 1 yellow and 1 purple (to give equivalent fractions).

Activity Six: Fraction Estimation using half as a benchmark. In this lesson learners used $\frac{1}{2}$ as a benchmark. I demonstrated a fraction concept by using beans, by putting 3 lots of 2, looking at a fraction as part of the whole collection of beans. White beans were used to represent any fraction (on the Right Hand Side), but brown beans on the Left Hand Side represented $\frac{1}{2}$ (benchmark). Beans were used to show fair shares. I encouraged learners to use verbal language as a way of describing what the pictures convey.

Activity Seven: Fraction Estimation using mental images. I asked the learners to picture fractions in their minds and to order them from the smallest to the largest e.g. $\frac{9}{16}$, $\frac{8}{9}$ and $\frac{1}{14}$. The solution is $\frac{1}{14}$, $\frac{9}{16}$ and $\frac{8}{9}$. I told the learners that the first fraction is close to 0, the second fraction is close to $\frac{1}{2}$ and the third fraction is close to 1 whole.

Activity Eight: Use mental images to circle larger fractions. I first demonstrated this lesson using fraction circles e.g. which fraction is larger between $\frac{2}{3}$ or $\frac{2}{4}$, $\frac{3}{6}$ or $\frac{1}{2}$ or $\frac{4}{8}$ or $\frac{6}{8}$, to revise what we did before. “Now class, no concrete teaching and learning materials are allowed, but you must use your mental images to figure out the following fractions such as $\frac{8}{16}$ or $\frac{13}{16}$ ”. I asked the learners to circle the larger fractions.

Activity Nine: Mixed and Improper fractions. The learners were asked to move from mixed numbers to improper fractions and back into mixed numbers. The objective of this lesson was to show the understanding of mixed number and improper fraction concepts, and learners

were asked to draw the pictures to convince me that they understand these two concepts. The learners were further asked to explain their pictures.

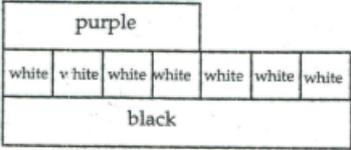
Activity Ten: Bingo Games For Bingo games, learners were asked to put x on all fractions on a given grid ($\frac{2}{4}, \frac{3}{12}, \frac{6}{12}, \frac{4}{12}, \frac{3}{6}, \frac{10}{12}, \frac{6}{6}, \frac{1}{12}, \frac{2}{3}$) that are equivalent to $\frac{1}{2}$ and $\frac{5}{6}$. For learners to tackle the above fractions, they needed to know the concept of equivalent fractions and to simplify the given fractions to resemble the desired solutions.

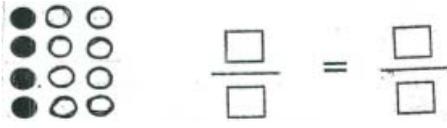
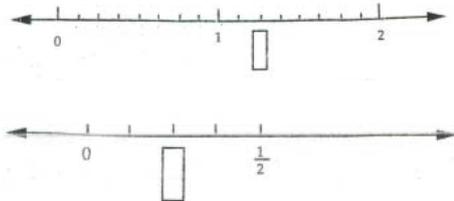
3.4 OTHER DATA SOURCES

As well as the data generated from the RNP activities, data sources included fraction test, learner interviews, Mathematics examination, English examination and my research diary. These are discussed in turn in the following section.

Fraction Test. Following each two week cycle of teaching, the learners wrote a test on the fraction content covered. The fraction test had thirteen questions that were marked out of twenty marks. The questions are set out in (Kilpatrick et al., 2001) together with a brief rationale to relate the questions to the mathematical proficiency strands.

Table 3.3 Description and classification of fraction test items

Item	Question	Rationale	Strand of proficiency
1 -3	Fill in > or < or = $\frac{3}{10}$ ----- $\frac{1}{2}$ $\frac{1}{2}$ ----- $\frac{6}{11}$ $\frac{3}{7}$ ----- $\frac{1}{2}$	Questions 1, 2 and 3 dealt with inequality signs. These questions had $\frac{1}{2}$ as a benchmark, so the learners were to say whether a fraction is greater, less or equal to half.	Conceptual understanding
4	Look at the picture. Answer the following questions  Purple is what fraction of the black rod? White is what fraction of the black rod? Name the fraction shown by the whole above picture.	Questions 4.1, 4.2 and 4.3 had pictorial representation of a mixed number. These questions were taken from activities 1, 2, 3 and 4 where the black rod was used as a train. Learners were asked to compare white and purple rods with the train.	Procedural fluency

Item	Question	Rationale	Strand of proficiency
5	<p>What fraction of the set is shaded? Name the amount in two ways.</p> 	Question 5 tests the recognition of a fraction when presented as a part of a set of discrete objects	Conceptual understanding
6	<p>Imagine that you have a bag of rods to work with. You are to show $\frac{2}{3}$ in two different ways. Draw pictures to show that you can do this.</p>	These questions goes back to the concrete materials and test the ability to model a fraction.	Procedural fluency
7, 8	<p>Write the number that should go in the box on this line</p> 	Questions 7 and 8 focus on the idea of a fraction on a point on the number line and also on ordering fractions	Conceptual understanding
9, 10	<p>Draw a picture to show $\frac{1}{2}$ and $\frac{2}{4}$. (Use < ; > ; =)</p> <p>Draw a picture to show $\frac{4}{9}$ and $\frac{1}{3}$. (Use < ; > ; =)</p>	Questions 9 and 10 test the ability to model fractions in order to compare their relative size.	Procedural fluency
11,12	<p>a) Circle the larger fraction. Circle both fractions if equal.</p> <p>b) Write a reason for each answer.</p> <p style="padding-left: 40px;">$\frac{8}{9}$</p> <p style="padding-left: 40px;">$\frac{3}{2}$</p>	Questions 11 and 12 dealt with comparing fractions, and also to give the reason why a fraction is greater or less than the other. These questions tested the learners' ability to reason their answers.	<p>a)Procedural fluency</p> <p>b)Adaptive reasoning</p>
13	<p>Show how 8 chocolates can be shared among 5 children</p>	Question 13 was a word problem. This question asked for a strategic competence from learners. It was a non-routine question.	Strategic competence

The fractions test covered all the five content strands of mathematical proficiency, but was largely focused on conceptual understanding and procedural fluency. The classification of question items is not always easy but was guided by the descriptions in Table 3.4.

Once the tests were marked, the marks for each question were entered onto a spread sheet and the scores for each strand of proficiency was computed. This analysis was planned to reveal the mean performance of the class groups in the items linked to each strand and the extent to which each strand of proficiency was developed.

Table 3.4 Identification of proficiency strand to each test item

Strand	Description	Examples	Questions	% of test marks
Conceptual understanding	the integrated and functional grasp of mathematical ideas	Mathematical vocabulary Modelling	1, 2 3 5 6,7 8,	37%
Procedural fluency	the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately	Choosing correct procedure; Practice of routine examples Using a variety of methods	4.1, 4.2, 4.3, 9, 10, 11a, 12a	37%
Strategic competence	the ability to formulate, represent, and solve mathematical problems	Non routine problems	13	16%
Adaptive reasoning	the capacity for logical thought, reflection, explanation, and justification.	Explaining answers	11b 12b	10%

Learner Interviews. Cohen, Manion and Morrison (2011) say the strengths of structured interviews are that the researcher has control over the topics and the format of the interview. This is because a detailed interview schedule is used. Consequently, there is a common format which makes it easier to analyse, code and compare data. The researcher used both purposive and stratified selection methods in selecting the learners that will be interviewed. In purposive sampling the researcher handpick the cases to be included in the sample on the basis of their judgement of their typicality. In this way they build up a sample that is satisfactory to their specific needs (Cohen, Manion & Morrison, 2011).

I used a combination of both purposive and stratified selection methods to select learners (N=12, see Appendix C) for focus group interviews. Four learners were chosen from each of the three classrooms according to their performance in the fraction test. The group

was heterogeneously mixed, six girls and six boys of different abilities. Purposive selection methods involve the researcher handpicking the cases to be included in the sample on the basis of typicality and for the specific qualities they bring to the study (Cohen, Manion & 2007, Lankshear & Knoebel, 2004

For the study I chose learners who articulate in English, who were active in the RNP activities, who achieved high, medium and low marks in the fraction test. The size of the sample was considered sufficient as no generalization on the entire population was sought.

The interview was structured around the following fraction problem: I had all the kinds of concrete teaching and learning materials such as beans, fraction strips, and fraction circles on my table for learners to see. I was at pains to make sure that all the twelve learners understood the questions and so I read them out and explained in isiZulu. During the interview learners were to choose the concrete teaching and learning materials to work with. When they were to divide the chocolate amongst the five friends, they were to split it practically and use terms such as break, cut or divide.

Ntombi, Thandi, Pinky, Gugu, and Thobile have 8 bars of chocolate that they want to share equally among the 5 of them so that nothing is left. How many pieces of chocolate will each child get?

After giving the learners a chance to write their answers, the following prompts were used:

1. Which concrete teaching and learning materials can you use to model this problem?
2. Why do you choose these concrete teaching and learning materials?
3. How many chocolates will each child get?
4. How many chocolates are left over?
5. How will you share the 'left over' chocolates amongst the 5 children?
6. Now how many chocolates will each child get?
7. Altogether, how many chocolates did each child get?

Kvale (1996) says interview transcripts provide powerful evidence for presenting data and making conclusions. Group interviews save time and are realistic in classroom contexts. Interviews can provide unexpected but useful perspectives. The learners wrote their responses on the interview schedule that was provided by me. The learners' transcripts were collected and were analysed by me.

All the learner research data was collected in 2009 October to November in a school in the Pinetown District. Conducting interviews is more time consuming than using

questionnaires. Another challenge was in the English language because all the learners speak isiZulu as a home language. I had to explain the interview questions in both English and isiZulu, in other words, used code-switching.

Mathematics Examination Marks. The mathematics examination marks were used in this research to check for the reliability of the learners' marks in mathematics. The mathematics examination papers are set by the districts Department of Education, and they are standardized.

English Examination Marks. The English examination marks were used in this study to test the literacy proficiency. Mathematics is taught and assessed in English in the school where the study was conducted, but I used code-switching to explain fractions concept. The governing body, the principal and the parents of this school chose English as the language of learning and teaching (LoLT). The majority of the parents perceived English as the gateway to global opportunities and therefore wanted their learners to participate in their education through the medium of English. My study confirms that a good performance in English facilitates a good performance in Mathematics.

Research Diary. According to Cohen, Manion and Morrison (2011) observational data are attractive as they afford the researcher the opportunity to gather 'live' data from 'live' situations. The research diary was written daily, to capture the moments as they were actually happening.

3.5 ETHICAL MATTERS

Issues of ethics are central to the entire research process (Cohen, Manion & Morrison 2011). I ensured that ethical issues were observed throughout the process. One of the key ethical considerations was the issue of anonymity and confidentiality (Henning, 2004; Cohen, Manion & Morrison, 2011). I assured participants of anonymity by protecting their identities and ensuring the maintenance of confidentiality (Burns, 2000). The learners who participated in the study participated of their own free will and with their parents' consent. I applied for an ethical clearance certificate from the University of KwaZulu-Natal. I wrote the consent letters to the school where the study was going to take place (principal, school governing body and teachers), the parents of minor learners to inform them about the nature and the

importance of the study. Copies of these letters are found in Appendix B. I explained in the consent letters about the use of pseudonyms to protect the anonymity and the confidentiality of the participants, school and teachers. The participants in this study are the learners in my classes and so the relationship is definitely unequal, but extreme care was taken to treat all learners fairly and ensure that all learners received quality teaching.

The University of KwaZulu-Natal applied for permission from the Department of Education in the province of KwaZulu-Natal, the department granted us permission to carry out the study. Then the university gave me the Ethical Approval Number: HSS/ 0743/ 2009: Faculty of Education (see Appendix B).

3.6 SUMMARY

This chapter gave a detailed description of how the study was conducted. It covered items such as RNP activities, learner interviews, research diary notes, the fraction test written by learners and Mathematics and English examination marks. In the following chapter, the results will be presented.

CHAPTER 4 RESULTS

In this chapter I relate what happened as I taught the described lessons to 7A, and in the process what I learned, the thoughts I gained and how I was able to change my teaching for 7B. Similarly I relate the experiences I had with teaching 7B and the changes that were made for 7C. My thinking and development was informed by my research diary, peer input, analysis of RNP activities, the fraction test, learner interviews, Mathematics and English Examinations. Each of these is discussed below.

4.1 FORMATIVE EXPERIENCES DURING THE ACTION RESEARCH CYCLES

Here I discuss each of the cycles in turn. For each of them I relate what happened with particular reference to the critical moments that caused me to reflect and make changes for the following cycle. Critical moments are unforeseen developments that typically provide heightened opportunities for insight and changes (Mc Niff, 2013).

Cycle 1 (7A). Ten activities selected for the challenges and learning they produced are described here.

Activity 1: Familiarisation with the Cuisenaire rods. Activity 1 was designed to introduce the learners to the use of Cuisenaire rods and it served to introduce fractions as different from whole numbers. Longer rods were used as units and shorter rods used as the fractions of the longer rods. When the learners tried to do Activity 1, which involved making trains of several rods to be equal to identified rods, the unfamiliarity with the materials led some of them to try to arrange the rods vertically instead of flat on the desk. Learners appeared to be somewhat nervous and excitable when they were doing this activity. As a facilitator, I told them to calm down and focus on the activity. I realised that the novelty of the brightly coloured rods was a distraction. This was a critical moment.

On reflection, and for the next two cycles, I decided that more teacher demonstration on the use of the Cuisenaire rods would lead to less confusion. I thought perhaps the learners were nervous to use the new materials and anxious about breaking them, I decided to try to put the next group at ease by explaining that the materials to be used were strong.

Research Diary

The research diary for this study had information about what actually happened inside as well as outside the classroom throughout the duration of the study. Information about the RNP activities that were completed by learners, what went wrong i.e. did learners understand what was expected of them to do or what went right i.e. were the learners able to accomplish the activities with ease. There are also some reflections from my critical teacher's observations during and after the lesson, such as whether the learners used the concrete teaching and learning materials, and which concrete teaching and learning materials were used to work on RNP activities.

I had the opportunity to discuss the development of my research on an on-going basis with my supervisor and recorded her suggestions in my diary. There was also some advice on how to structure the fraction test in order to answer the research questions. There was an interview schedule with responses from the focus group. There was also a timetable for the study and the observation dates for my critical teachers. Formative comments on individual lessons are included with the accounts of the lessons.

Critical friends' input

My critical friend teachers, Jabu and Ben, observed three lessons (one for each class) and gave me reflections after lessons. Their input is discussed in Table 4.1 below:

Table 4.1 Summary of critical friend input into the teaching in each cycle

	Date /Time	Situation	Participants	Action Observed	Reflection
Cycle One Gr 7A Jabu	October 14 9h00- 10h00	Inside Mr Nyathi's classroom	Mr Nyathi Grade 7A learners	Learners sitting in groups Mr Nyathi teaching equivalent fractions Learners completing RNP activities Mr Nyathi offers guidance.	Mr Nyathi believes in co-operative groups. Acts as a facilitator
Cycle Two Gr 7B Ben	October 28 9h00- 10h00	Inside Mr Nyathi's classroom	Mr Nyathi Grade 7B learners	Mr Nyathi handing out worksheets Learners sitting in groups. Learners filling in worksheets. Mr Nyathi assisting groups with problems	Co-operative grouping. Learners actively involved. Constructivism.

Cycle One Gr 7C Jabu	November 13 9h00- 10h00	Inside Mr Nyathi's classroom	Mr Nyathi Grade 7C learners	Desks facing the front. Mr Nyathi handing out fraction test.	Fraction test important. Discipline maintained.
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I began the project with grade 7A, and taught ten sixty-minute fraction lessons as outlined in the school time table. I divided the class into six groups of eight learners and learners worked together to complete fifteen selected RNP activities. My critical teacher (Jabu) observed one lesson with grade 7A and gave me a feedback after the lesson. Jabu said, "Learners will always be involved if they are working in groups". After each lesson, I marked their completed RNP activities. I met with my critical teachers and looked at the marked RNP activities together and discussed the success / failure of the lessons. After the completion of the ten fraction lessons and fifteen RNP activities I gave grade 7A a fraction test to be written individually, and I marked the test.

The second group was grade 7B; the teaching sequence began all over again as I began the project with grade 7A, and taught ten sixty-minute fraction lessons as outlined in Time table. I divided the class into six groups of eight learners and learners worked together to complete fifteen selected RNP activities. My critical teacher (Ben) observed one lesson with grade 7B and gave me a feedback after the lesson. Ben said, "The learners must finish one activity first and collect all the worksheets before giving them another worksheets to complete. Give learners some extra time to finish activities". After each lesson, I marked their completed RNP activities. The critical teacher (Ben) and I looked at the marked RNP activities together and discussed the success / failure of the lessons. After the completion of the ten fraction lessons and fifteen RNP activities I gave grade 7B a fraction test to be written individually, and marked the test.

For grades 7B and 7C I included a lesson on folding a paper strip to remedy the confusion of the whole number concept with the fractions concept. Learners were given a 20 cm long paper strip and I asked them to fold it into two equal pieces and asked them guiding questions. "How many pieces are there? How many folds are there? What do we call each piece? Show me one half of the strip? How many halves are in one whole? In this cycle the fraction language was emphasized.

This cycle was continued the following week, I taught ten sixty-minute fraction lessons, fifteen RNP activities with grade 7C and a fraction test at the end. The other critical friend

(Jabu) and I began the project with in grade 7A, and taught ten sixty-minute fraction lessons as outlined in Time table. I divided the class into six groups of eight learners and learners worked together to complete fifteen selected RNP activities. Next time in cycle three, I remedied the lesson on Equivalence and by asking learners to order fractions with the same numerators and different denominators symbolically. To complete this activity, they must draw on experiences from their mental images. This activity was to reinforce the idea that the more parts a unit is divided into, the smaller each part is. Jabu observed one lesson with grade 7C and gave me a feedback after the lesson. Jabu said, “This grade 7C seem to complete all the activities quickly, so I think they understand the fraction lessons and activities better than grades 7A and 7B”. “I think they are the brightest stars in this study”. The main value of the critical teachers was the support they gave me in discussing the work as the action research cycles preceded.

Activity 2: Modelling fractions with Cuisenaire rods.

Given the unit learners were asked to find the fraction names for the other rods. In Grade 7A, all six groups had the concept of a fraction being part of a continuous whole correct as evidenced by their drawings of $\frac{3}{5}$ using two different units (see Figure 4.1). They were not used to identify the unit, but rather were seen as a part to be iterated to make the whole. This is clear from the obvious use of the rods as a template to draw the whole, as seen in Figure 4.1. This confirmed for me that the learners understood how to use the rods; they had sufficient explanation and practice. I therefore maintained this in the following cycles.

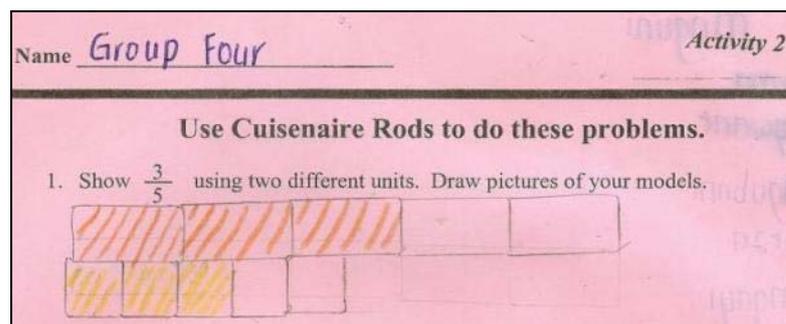


Figure 4.1 Class 7A Group Four: Depiction of $\frac{3}{5}$ using Cuisenaire rods

Activity 3 and Activity 4 were completed smoothly and achieved the desired outcomes without modification. Consequently no changes were devised.

Activity 5: Modelling fractions with fraction circles and other discrete objects. The original RNP activity called for the use of counters. We did not have counters at school and so I improvised with two colours of beans and they worked very well.

The fraction circles set used in this study consisted of six circles that showed whole, halves, thirds, fourths, fifths and sixths. The teacher set had magnetic strips on the back so that the pieces could be stuck to the chalkboard.

Fraction circles are normally used to engage learners in exploring and discovering relationships between the pieces, and sometimes to solve problems that require the four basic operations such as addition, subtraction, multiplication and division of fractions. Fraction circles may also be used to show fractions of different kinds.



Figure 4.2 Learners working with beans as discrete objects

Beans (brown and white) were used as the concrete teaching and learning materials to aid learners to see a fraction as part of a collection of objects. In this activity my learners confused the concept of whole number with the concept of a fraction. This could be seen as they represented, for example, $\frac{3}{5}$ as three white beans placed over five brown beans (see Figure 4.2). This was a critical moment and on reflection I decided that to remedy this confusion, in the next cycle, we would start by using paper folding.

Activities 6 to 10 did not produce any problems. In addition I did not analyse Activities 11 to 15 because all the eighteen groups fared well in all of them, there were no critical moments for me.

Cycle 2 (7B). In this section I will describe the changes made as a result of the experiences of Cycle 1 and other activities that provided critical moments for reflection. For ease of reading they are described in numerical order.

Activity 1: Familiarisation with the Cuisenaire rods. This was delivered with more attention to introducing the rods as planned, and I decided to try to put the group at ease by explaining that the materials to be used were strong. This reduced the amount of confusion and the lesson went more smoothly.

Activity 2: Modelling fractions with Cuisenaire rods. This time, with 7B, I spent time discussing what $\frac{3}{5}$ meant such as $\frac{3}{5}$ of a whole or 3 out of 5 equal parts, then I had the learners explore with the concrete teaching and learning materials to find pieces that fit together to make fifths. I also asked the learners to demonstrate various ways to model 3 out of 5. After the learners had explored with the concrete teaching and learning materials for several minutes I had every group chose one way to model $\frac{3}{5}$. At first they were not all able to do the task and produced responses as shown in Figure 4.3. This is a well-known error in which the numerator and denominator are depicted as whole numbers instead of fractions.

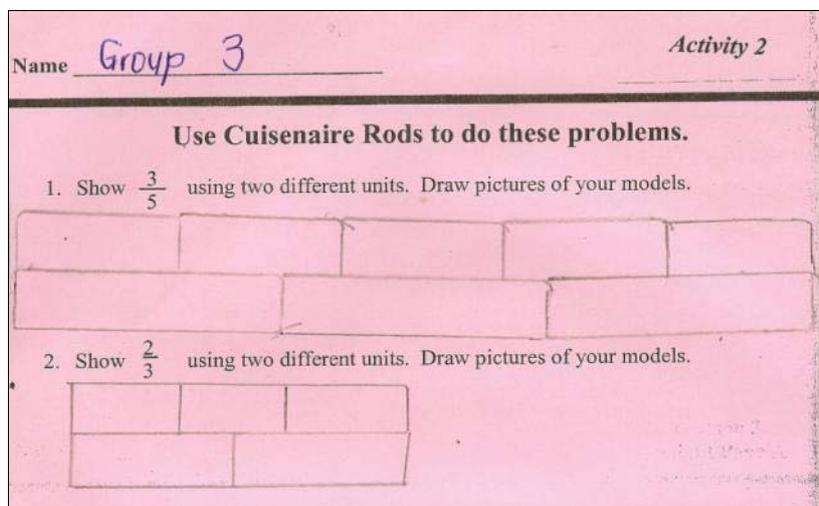


Figure 4.3 Response showing the numerator and denominator are depicted as whole numbers instead of fractions.

To remedy this and in Cycle 3 I used paper strips and number lines. One fold is equal to two halves, two folds are equal to four quarters etc., all folds were vertical so as not to confuse the learners. From the paper folding model we moved to different number lines to model different fractions. By using the number lines, learners were able to visualize that $\frac{1}{4}$ and $\frac{1}{12}$ are less than $\frac{1}{2}$ and that $\frac{1}{12}$ is smaller than $\frac{1}{4}$ since twelfths are smaller pieces than

fourths. In this activity learners were moving from mental images (pictures) to verbal symbols and eventually to written symbols and vice versa according to the Lesh Translation Model (see Figure 2.2).

Activity 3: Using Cuisenaire rods as Units. For Activity 3, Group 4 (see figure 4.4) had an interesting error and unusual orientation of the rods. They had the fractions correct, but the written symbol incorrect. I understood this to indicate that they were able to work with the manipulative, pictorial and verbal symbols of the Lesh model but struggled to grasp the symbolic representations. This was a critical moment for reflection and I decided that in Cycle 3 I would address this by again using paper folding and number lines in combination with naming and writing the fractions on the chalkboard.

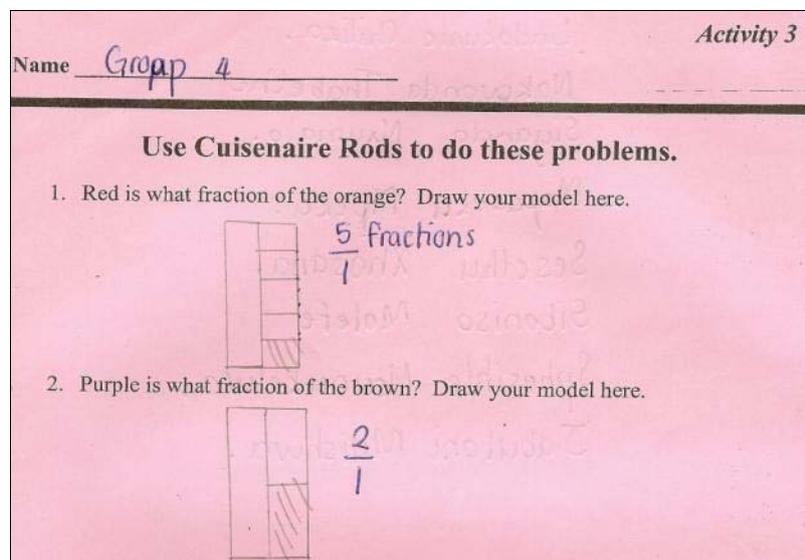


Figure 4.4 The pictorial representations are accurate but the symbolic representations are wrong

Activity 5: Equivalence and Cuisenaire rods. With the additional use of paper strips and number lines the lesson proved much more successful and the change was maintained for Cycle 3.

Activities 6, 7, 8, 9 and 10 did not cause any problems or critical moments for reflection. They were maintained for Cycle 3

Cycle 3 (7C). In this section I will describe the changes made as a result of the experiences of Cycle 2 and other activities that provided critical moments for reflection. For ease of reading they are described in numerical order.

Activities 5 to 8 and 10 were all completed satisfactorily with the planned revisions implemented.

Activity 9: Mixed and Improper fractions. Most groups, fared well on this improper fraction activity and the remainder completed it with limited assistance, indicating that the activities adopted from the RNP materials had allowed them to understand the concept of improper fractions and they were able to apply their knowledge correctly. Learners were able to make connections between pictorial and symbolic representations when they did this activity (Lesh Translation Model). Figure 4.5 shows accurately completed group work.

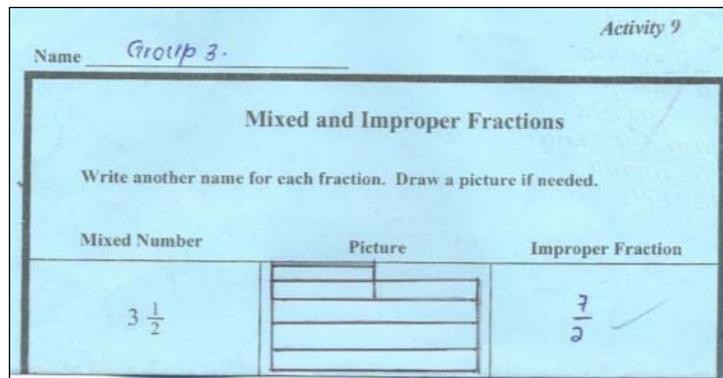


Figure 4.5 Example of learners' work in Activity 9

4.2 SUMMARY OF THE CHANGES INTRODUCED DURING THE ACTION RESEARCH PROCESS

At the end of teaching cycle 3 (7C) I arranged meetings with my critical friend teachers and my supervisor to review what had happened, what I had learned and how my teaching had changed during the process. My research diary was consulted for this purpose.

We were in agreement that the experience had been valuable for all concerned. From my point of view, as the educator, having the well-researched RNP materials as a basis for my planning gave me confidence but I also felt confident enough to make changes and adapt them to my own teaching conditions. I found that my understanding of my own teaching context was enhanced by looking at it through the lens of the Lesh model. I also realised that I was becoming more acquainted with the notion of mathematical competence as I saw my learners discussing and arguing and I recognised that my teaching had changed.

From the point of view of the learners themselves I had seen their involvement in the lessons increase as they got used to handling the range of concrete teaching and learning

materials available to them. I felt that they were beginning to see the materials as a tool to understanding fractions and working with them more effectively. It was relatively easy for them work with real life situations, concrete teaching and learning materials and verbal symbols but translating to symbolic representations was a significant challenge and they required more assistance.

Our discussions about the helpful changes that took place and helped to answer Research Question 2: Which pedagogical practices and strategies are helpful in developing mathematical proficiency in fractions? They are summarised by the following points:

- Adapting and adopting well researched materials is advisable; the RNP materials served their purpose as a framework for my teaching.
- If the learners do not have experiences with a range of concrete teaching and learning materials then the aids themselves can become a distraction and focus attention away from the desired learning. This was the case when I introduced them to the brightly coloured Cuisenaire Rods – they just wanted to play with them. I needed to acknowledge this and explain their use and purpose and allow time for this in lesson planning.
- I found that Cuisenaire Rods were not a stand-alone solution to problems, including that of confusing whole numbers and fractions, and the teaching was almost always more effective with the addition of folding paper strips and using number lines.
- I found it valuable to have teachers as critical friends to observe lessons and give feedback that would assist to improve the teaching and learning of fractions.

4.3 ACHIEVEMENT IN THE FRACTION TEST

At the end of each two week cycle, the class wrote a summative test on fractions. The test was the same for all classes. The results of this test are discussed below, with comparisons between the mean marks of each class.

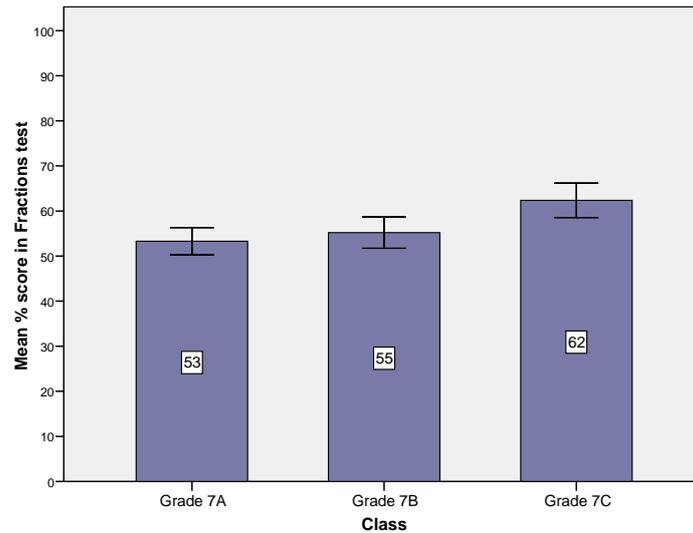


Figure 4.6 Mean % score on fraction test for each class

Figure 4.6 shows the mean % score on the Fraction test the three classes, grade 7C has the highest overall level mean score, followed by grade 7B and then 7A. Grade 7A was the first group to undergo a research study, grade 7B was the second group to be researched and grade 7C was the last group to be researched. I can infer that grade 7C understood the work better than grades 7B and 7A.

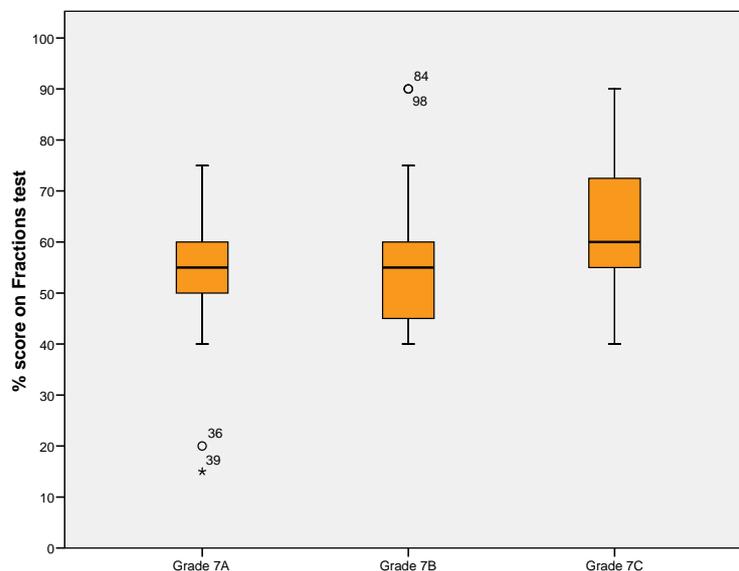


Figure 4.7 Distribution of marks within each class in the fraction test

A box plot showing the distribution of marks within each class is given in Figure 4.7. According to the inter quartile ranges, the distribution is widest in grade 7C, followed by grade 7B and grade 7A. The distributions within each class are fairly symmetric in grade 7A because the box is fairly evenly spaced between the whiskers. Because the 50th percentile

line is not evenly spaced in the box for grade 7C, but it is for grade 7A, the distribution is slightly more skewed in grade 7C, but it is not severely skewed. The distribution is more severely skewed in grade 7B.

A one way ANOVA test was used to determine whether there was a significant difference between the mean scores of the three classes in the fractions test. This showed that there was a statistically significant difference at the $p < 0.05$ level between the mean scores of Grade 7A and Grade 7C ($p = 0.001$) and between the mean scores of Grade 7A and Grade 7C ($p = 0.001$). Grade 7C which was the group taught in the third cycle of the action research obtained a mean score that was statistically significantly higher than the previous two classes. Other grouping factors that were considered were gender and age. Having used selected concrete teaching and learning materials to develop proficiency in fractions as the researcher I wanted to know whether girls or boys scored higher marks. A t-test showed that the mean for the girls ($M = 58$) and the mean for the boys ($M = 56$) was not statistically significant at the 95% level ($p = 0.328$).

The Grade 7 learners' ages ranged from 11 to 17 years. The mean scores were highest for learners who were between 12 and 14 years old, with over age learners on average performing less well. However, an ANOVA test showed no significant difference in achievement on the fractions test between the different age groupings in Grade 7.

The fractions test covered all the five content strands of mathematical proficiency, but was largely focused on conceptual understanding and procedural fluency. The classification of question items was described in Chapter 3. Figure 4.8 shows the pattern of how the learners performed in four of the mathematical proficiency strands.

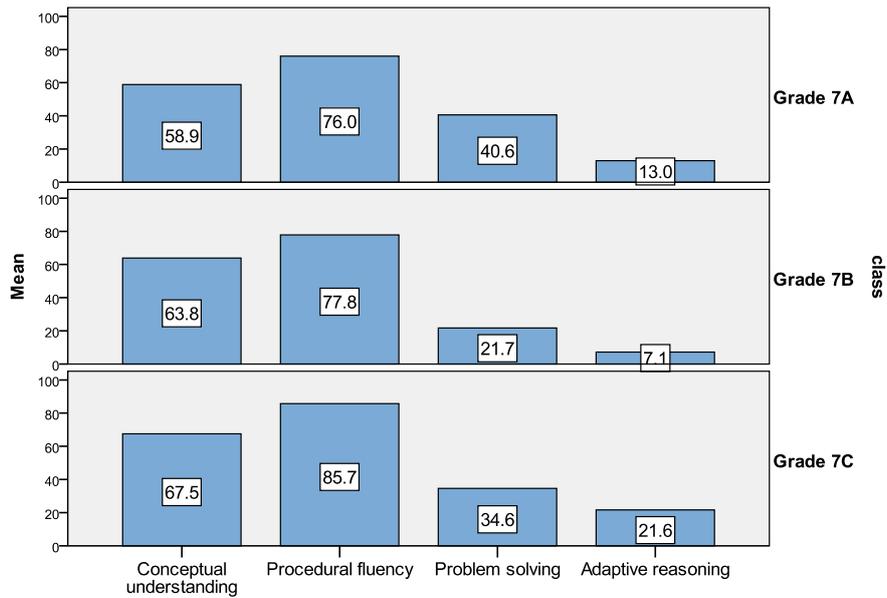


Figure 4.8 Mean % achieved in the fraction test for each strand of mathematical proficiency

Figure 4.8 shows that the achievement patterns were similar for all groups; their highest score was procedural fluency, they were carrying out procedures of fractions flexibly, accurately, efficiently and appropriately using the concrete teaching and learning materials. The second best category was conceptual understanding, they understood the concept of fraction through using both concrete teaching and learning materials and images. In problem solving Grade 7A was better able than Grades 7B and 7C. For adaptive reasoning all three groups were poor, they cannot tell why a certain fraction is greater than the other one.

4.4 LEARNER INTERVIEWS

Twelve representative learners completed a task and were asked questions about what they decided or thought and the reasons for their responses. The task took the form of a problem related to everyday life; in terms of the Lesh model it would be located in the Real life situation strand.

This was the question:

Ntombi, Thandi, Pinky, Gugu and Thobile have 8 bars of chocolate that they want to share equally among the 5 of them so that nothing is left. How many pieces of chocolate will each child get?

The learners wrote their responses on the interview schedule that was provided for the purpose. Out of seven questions asked in the interviews four questions were analysed for the benefit of the study:

Question: Which concrete teaching and learning materials can you use to model this problem?

The purpose of the first question was to see if the learners could select an appropriate concrete teaching and learning material for this problem presented to them. Fraction circles and Beans were provided and were equally popular.

Question: Why do you choose this/these concrete teaching and learning materials?

The reasons given were all very similar and included liking, knowing how to use or manipulate and being handy. The reasons the learners gave were not clearly linked to the problem they were dealing with, but seemed to be based on personal preferences. It is difficult to see how a fraction circle would be chosen in this case.

Question: How many chocolates each child would get, and how many are left over?

In response to this question, all twelve answered that each child would get one chocolate and there would be three left over. This led to the key part of the interview – the discussion of what to do with the three that were over.

Question: How will you share the 'left over' chocolate/s amongst the 5 children?

In terms of the mechanics of the dividing learners variously chose the verbs cut, divide or break. I did not investigate these choices but focussed on the results of the division. Ten learners described the products of the division as 'six halves' and two as 6 chocolate. I took this to indicate a high level of verbal proficiency in the area of fractions.

Question: How many chocolates will each child get now, and how much chocolate in total?

This question allowed the learners to show their capacity with symbolic representations and addition of fractions. Seven learners left the answer at $1 \text{ whole} + \frac{1}{2} + \frac{1}{10}$. They were able to compute $\frac{1}{2}$ divided by 5. The remaining five were able to add $\frac{1}{2} + \frac{1}{10}$ and give the answer $1 \text{ whole} + \frac{3}{5}$.

In order to get a clearer idea of the thinking of the learners and their use of concrete teaching and learning materials, an in depth personal interview was conducted with one learner, Bridget. She was chosen because she was an articulate and able learner. The full transcript is included here to provide some rich authentic data and to illustrate the thinking evident in the grade 7 learners.

Interview with Bridget

- Researcher So you know the work we were doing for ten days, I want us to take a closer look because some of it was interesting and we did not finish talking about it. I was just wondering what we were doing in mathematics for the past ten days.
- Bridget *We were learning about fractions.*
- Researcher Ntombi has 8 slabs of chocolate. She wants to share them with her four friends: Thandi, Pinky, Gugu and Thobile. How many pieces of chocolate will Ntombi and each of her friends get? You are free to use any concrete teaching and learning materials that might help you. Uyezwa (understand).
- Bridget *(Taking 8 rods) Ngicabanga ukuthi (I think) 8 rods will help me solve chocolate problem.*
- Researcher Yes, how would you share them with your friends?
- Bridget *I will first give each of my friends 1. (Giving out 5 Cuisenaire, and remaining with 3 rods).*
- Researcher And then.....
- Bridget *Ngingasika lama-rods awu-three asele to give 6 halves. Each of my friends will now get a half. (Giving out 5 rods and remaining with 1 half rod).*
- Researcher And now what are you going to do with that one half rod, remaining?
- Bridget *Now, I will also cut this remaining half to give 5 pieces. Now each of my friends will get a tiny piece ($\frac{1}{5}$).*
- Researcher So in all, what did each friend get? You can even draw a picture if you can.
- Bridget *Umngani uthole (each friend got) 1 whole + $\frac{1}{2}$ + $\frac{1}{5}$ of the 8 chocolates.*

Researcher Good, thank you, Bridget.

Bridget was able to move in between and among all five translations according to Lesh Translation Model (see Figure 2.2)

4.5 ACHIEVEMENT IN THE GRADE 7 MATHEMATICS EXAMINATION

This study focused on the teaching of one section of the curriculum, fractions, which is therefore, incorporated in the end of year examination. These papers are set by the local department of education and can, therefore be seen to be objective. The three Grade7 classes were mixed ability classes with the same ratio of boys to girls' similar language and cultural backgrounds and so I was able to assume a level of similarity that would allow me to compare the effectiveness of the fractions teaching received by 7A, 7B and 7C. During the study they received different remedial interventions for their fraction work because of their involvement in the action research process and the conscious effort on my part to improve the quality of the teaching. I assumed that 7B would have an enhanced experience compared with 7A and, similarly, 7C would have a more improved experience than the other two classes. Therefore, if there were any differences in the final examination results, all other things being equal, it would indicate a causal effect of the 7C experience. I used the examination results for this purpose see figure 4.9 that shows the mathematics examination results.

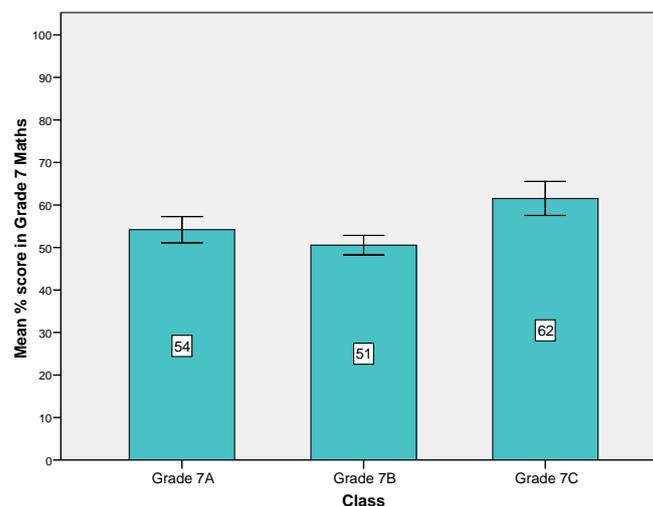


Figure 4.9 Mean marks for Grade 7 Maths examination, by class group

The overall mean score for the grade was 55.5%. Grade 7C has the highest mean score (61.5%), followed by the Grade 7A (54.2%) and then 7B (50.6%).

A one way ANOVA test showed that the variation between the groups was greater than the variation within the groups at the 95% level. That is we can be say with 95% certainty that the class grouping was a significant factor in the differences in the mean scores in the Grade 7 mathematics examination. A post hoc test showed that Grade 7C had a statistically significantly higher mean score than the other two groups. The details of this analysis can be found in Appendix D.

I interpreted these results to mean that the enhanced experience of 7C could have been a causative factor in their higher mathematics examination marks.

4.6 ACHIEVEMENT IN THE GRADE 7 ENGLISH

The role of language in learning has been discussed in the literature review in Chapter 2. In order to investigate the possibility that the higher scores in the mathematics tests and examinations obtained by Grade 7C could be attributed to their better command of English, a comparison of the English marks across classes was done.

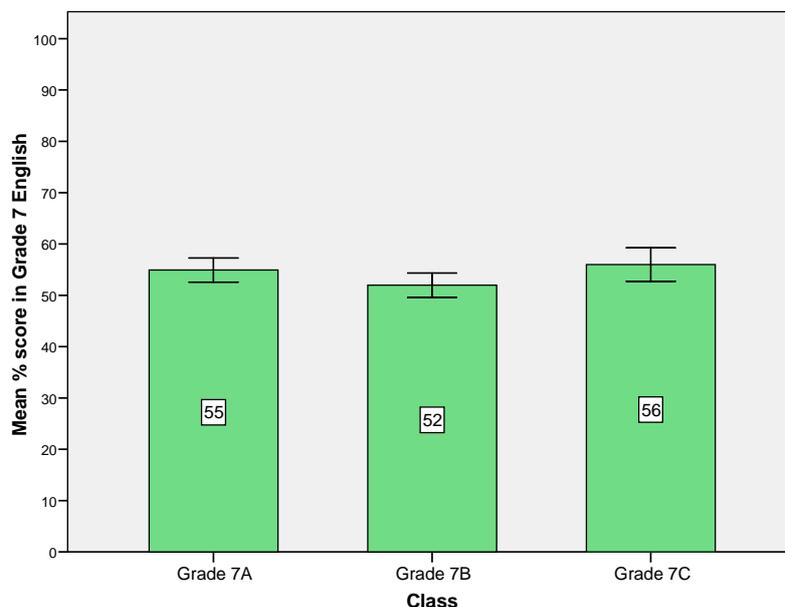


Figure 4.10 Mean marks for Grade 7 English examination, by class group

As illustrated (see figure 4.10) the mean % score in English examination was highest for Grade 7C, followed by the grade 7A and then 7B.

A one way ANOVA test was used to determine whether the difference between the mean scores of the three classes in the English examination was statistically significant. This showed that there was no statistically significant difference at the $p < 0.05$ level between the

mean scores of the groups. In other words, the difference in the means could be not be attributed to the class groupings with 95% certainty ($p = 0.96$) due to the great variation within the class groups. I cannot therefore infer that grade 7C has a higher command of English language than grades 7B and 7A.

4.7 SUMMARY

The summary of this chapter serves to confirm that the data obtained from the different data sources such as research diary, peer input, formative experiences during the action research cycles, summary of the changes introduced during the action research process, achievement in fraction test, learners interviews, mathematics and English end of the year marks were valuable sources for answering the research questions. In the following chapter, the discussion and recommendations will be presented.

CHAPTER 5 DISCUSSION

In this section I will discuss the findings presented in Chapter 4, in the light of the literature review and the frameworks of Lesh Translation Model, the five strands of mathematical proficiency and the reflections on the action research. This will provide answers to the research questions that guided this study:

Research Question One: Can selected concrete teaching and learning materials help in developing proficiency in fractions in Grade 7 learners?

Research Question Two: Which pedagogical practices and strategies are helpful in developing mathematical proficiency in fractions?

5.1 AN OVERVIEW OF THE STUDY

In Chapter 1, I provided the orientation of the research topic, the rationale and the purpose of the study as well as the critical questions. The main aim of the study was to find out whether concrete teaching and learning materials can assist in the learning of fractions and to investigate the pedagogical practices and strategies that are helpful in developing mathematical proficiency.

Chapter 2 was dedicated to the literature review. The review began with the learning of mathematics moving from the behaviourist views where learners are seen as absorbing mathematical ideas as the teacher present them, to constructivism where learners were seen as creating their own knowledge through their experiences of the world. The literature covered aspects such as the Lesh Translation Model, bridging from concrete to abstract, the South African Revised Mathematics Curriculum, Australian and Canadian Mathematics Curriculum, The South African Language Policy for education (LoLT), the Rational Number Project (RNP) and Kilpatrick's model of Mathematical Proficiency.

Chapter 3 provided a comprehensive discussion of the theoretical and conceptual framework, research design and methodology that was employed in order to generate data for the study. This chapter related the criteria used in the selection of the school that formed the sample. Action Research as the major modality was justified as the inclusion of quantitative data and the subsequent description of the research as adopting a mixed methods approach

The focus of Chapter 4 was on the analysis and interpretation of data. I gave an account of the three action research cycles and the changes made during the process. This was followed by analysis of the fraction test results and comparison of these with the end of year examination results (Mathematics and English). I concluded that, since the end of year Mathematics results for the last class to be taught (7C) were statistically significantly higher than the other two classes and there was no evidence that other factors correlated, their enhanced learning experience was a factor on the achievement of 7C.

This process of analysis was verified against the backdrop of the literature review in Chapter 2. The exploration of the data sources assisted me in gaining a deeper understanding of the multiple use of concrete teaching and learning materials as the basis for proficiency in mathematics.

The next two sections of this chapter discuss the answers to the research questions first, the role of the concrete materials is discussed using mathematical proficiency as a framework secondly, and the pedagogical practice of using different models of fractions is discussed using the Lesh Translation model as a framework. In both cases I refer to my action research experiences.

5.2 CAN SELECTED CONCRETE TEACHING AND LEARNING MATERIALS HELP IN DEVELOPING PROFICIENCY IN FRACTIONS IN GRADE 7 LEARNERS?

This research indicates that selected concrete teaching and learning materials can help in developing proficiency in fractions in Grade 7 learners due to their good marks in the fraction test. The five strands of proficiency were discussed in Chapter 2 and are now linked to the work done by the learners.

Conceptual understanding, refers to the understanding the ‘big picture’ of mathematics. Comprehending concepts such as fractions, that need to be applied to have an understanding of the ‘how’ and ‘why’ of mathematics. Conceptual understanding supports retention and prevents common errors. The learner must *understand* what is being done (Kilpatrick et al., 2001). Figure 5.1 shows an example from the test where the learner had to draw a picture to show the relationship between $\frac{1}{2}$ and $\frac{2}{4}$. These learners understand that the fraction can be represented symbolically in different ways?

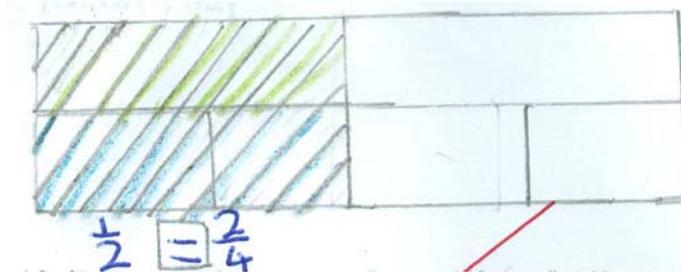


Figure 5.1 Test Example showing conceptual understanding

The mean score of the three classes in the items developing conceptual understanding was 63.4%. The learners benefited through the use of concrete teaching and learning materials and were able to move in between and among the five translations of the Lesh Translation Model. They were also able to solve fraction problems in the RNP activities.

Procedural fluency is being able to actually execute mathematics problems. This includes knowing what concepts to apply and how to apply them to solve a mathematical problem. The learner must *apply* what has been learned (Kilpatrick et al., 2001). The test items classified as developing procedural fluency were generally well done with the mean score for all three classes being 79, 8%. The mean for Class 7C which had the benefit of the improved teaching as they were the third cycle of the action research had a mean of 85, 7%. This high score suggests that this aspect of mathematical proficiency was developed by the use of the concrete teaching materials.

The concrete model is used to develop the algorithm which is then practiced in the RNP activities (*procedural fluency*).

Strategic competence is the ability of a learner to understand that there may be multiple ways to solve mathematics problems. A learner knows they can represent the problem and understand different ways it may be solved. The learner must *reason* about what has been done (Kilpatrick et al., 2001). This is closely linked to problem solving competence. The mean score for this strand in the fraction test was 32, 3% which is relatively low. However there was just one item so we cannot assume that this strand was poorly developed.

Adaptive reasoning addresses the discussion of how a problem is solved. This goes beyond memorization, but actually talking about why a solution worked. Two problems 11 and 12 on the fraction assessment, where they say circle larger fraction: circle both fractions if equal. Write a reason for each answer. These problems require learners to think logically, reflect, explain and justify. When recording the marks for each question, it was evident that

although many learners could correctly identify the larger fraction, very few were able to provide a sensible reason as shown in Figure 5.2. They were able to say that $\frac{8}{9}$ is smaller than $\frac{11}{12}$. They did not understand that the symbol $<$ means less than, because the reason given was “greater”.

11.

$$\frac{8}{9} < \left(\frac{11}{12}\right)$$

Greater

Figure 5.2 Example showing poorly developed adaptive reasoning

Productive disposition is a key element in being mathematically proficient. This means that a learner knows they have the ability and desire to learn, they see the value in learning and understanding mathematical concepts and strategies. The teacher must be able to create the environment in which learners want to engage with mathematics, an environment in which learners are expected to work hard to take risks and struggle (Kilpatrick, et al., 2001). This is not easy to measure, but it seemed to me that the learners enjoyed the lessons with the concrete teaching materials and this increased their interest in mathematics.

For this study on fractions I concentrated on the four strands: namely conceptual understanding, procedural fluency, strategic competence as well as adaptive reasoning: although all five strands (see Figure 2.1) were treated in this research. The learners scored highest in procedural fluency followed by conceptual understanding then problem solving and were very poor in adaptive reasoning. Learners were not able to dispel the perception that school mathematics is unrelated to a learner’s everyday experience.

5.3 WHICH PEDAGOGICAL PRACTICES AND STRATEGIES ARE HELPFUL IN DEVELOPING MATHEMATICS PROFICIENCY IN FRACTIONS?

The useful role of concrete teaching and learning materials has been discussed above, and the argument that the use of concrete materials is a helpful pedagogical practice in developing mathematical proficiency in fractions has been made. In this section, I will discuss the emphasis on different representations found in the RNP activities, using the Lesh translation model ((Lesh et al., 2003) as a framework (see Chapter 2.2.1 for a full description of the model). I will use illustrations from my action research where appropriate.

The five types of representation used in the Lesh model are: real life, oral, pictures, concrete apparatus and written symbols. Here I present a case for considering the Lesh translation model as a guide for mathematics proficiency. Building on the theories of Piaget, Bruner and Diennes, the Lesh model suggests that mathematics proficiency can be developed by involving learners in activities that embed the mathematical ideas to be learned in five different modes of representation with an emphasis on translations within and between modes. The Lesh model was used in developing the RNP fraction curriculum that was used in this study. Classroom teachers, such as the researcher in this study, can use the Lesh translation model to guide how they implement RNCS for Mathematics (Grades R – 9) in their classrooms. The power of the Lesh translation model can be seen in its multiple uses such as: a model for curriculum development, a model for classroom curriculum decisions and a model for assessment.

This study focused on the conceptual understanding of fractions, equivalent fractions, comparing fractions, and the addition of fractions. I did fifteen RNP activities with the learners during the study. The RNP researchers have sequenced fraction lessons, fractions are introduced as one representation with the translation to another representation. My learners, through the use of RNP programme, had an opportunity to explore fraction concepts by using multiple representations in the form of fraction circles, pictures, story problems and written symbols. The activities done in the two weeks of teaching and learning and in the subsequent test can be identified as stages of the translation model. The problem that occurred in the test and also in the interview is used as an example.

Ntombi, Thandi, Pinky, Gugu and Thobile have 8 bars of chocolate that they want to share equally among the 5 of them so that nothing is left. How many pieces of chocolate will each child get?

The story context is a real life situation. Learners could use the Cuisenaire rods or fraction strips as concrete teaching and learning materials (manipulates) to model the situation and seek a solution. In order to record the thinking and the solution, pictures are drawn.

Thereafter, the learner would either use verbal symbols to speak out the solution or put the written symbols down on paper. This can be seen in the work of Bridget during the interview. The participant was able to give the reasonable answer using visual aids, such as, drawing a picture and then numbers which are symbolic. According to Lesh model, Bridget was able to move from the real life situation (chocolate), manipulative (Cuisenaire Rod), pictures (when

drawing one whole, and three wholes divided into fifths, verbal symbol (one whole, one fifth and one fifth and one fifth) and written symbols ($\frac{8}{5} = 1$ whole and $\frac{3}{5}$).

Language plays a part in the Lesh Translation model when learners try to express the concept of fraction verbally. Mathematics is taught and assessed in English in the school where the study was conducted. I allowed the participant to code switch, so that she can answer her thoughts freely and confidently using both English and the mother tongue, which is isiZulu. Setati and Adler (2001) said code switching is a communication model that is used in mathematics classrooms where the language of learning and teaching (LoLT) is English. A teacher code switches in order to translate or clarify instructions and also to explain mathematics concepts in the learners' vernacular language for proficiency in mathematics.

During the group interviews on sharing chocolates, although learners (both boys and girls) were able to move in between and among the translations: only a few learners were able to reach the written symbol translation. Now I can strongly say that teaching mathematics is no longer a transfer of mathematics content and skills (Behaviourism), but an active participation of both learners and teachers (Constructivism). A teacher can also move from constructivism to modelling, with the aid of RNP activities such as the ones that I implemented in this study.

There are examples of learner work (see Figure 5.3) where the learners had correctly used the concrete materials, and drawn the pictorial representation correctly but were unable to provide the correct written symbol. The pedagogical practice of emphasising different representations could help develop competence in this area.

The use of the multi-representation approach in the RNP activities is a pedagogical practice that I consider helpful in learning fractions. The bottom line of RNP activities was to dispel incorrect perceptions that school mathematics is unrelated to a learner's everyday experience. They developed language necessary for meaningful communication in the mathematics classroom. They also developed problem – solving skills, confidence and enjoyment of mathematics.

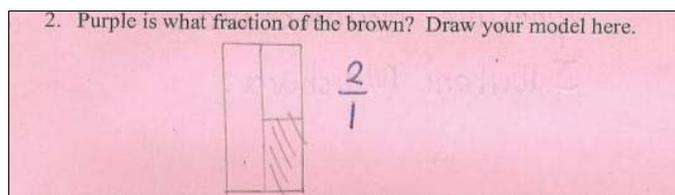


Figure 5.3 Correct Representation of fraction but incorrect written symbol

The RNP activities were suitable for the level of the Grade 7 learners in this study. Reys, et. al. (1992) say that learning activities that fall within the learner's grasp are identified as being within a child's zone of proximal development. Research supported that learning activities that fall within a child's zone of proximal development have a high probability of success, thereby contributing to the child's mathematical growth and development.

5.4 REFLECTIONS ON THE ACTION RESEARCH CYCLES

The results presented in Chapter show that the marks of the third class taught, Class 7C were statistically significantly higher than the other two classes for both the fractions test and the final Grade 7 Mathematics examination. Analysis of the other marks available, the Grade 7 English examination showed no statistically significant difference between the classes. Likewise, age and gender were not significant grouping variables. Since the only significant difference was in Mathematics where the research intervention was conducted, it appears that the learning was improved by the time the third cycle of teaching was conducted.

Through the study I learned to set the mathematics test following the Kilpatrick's mathematical proficiency strands (see figure 2.1). In future I will use it whenever I set and analyse mathematics tests. I also learned statistics, ANOVA and how to analyse the results. I found it difficult to keep the research diary because I was in the habit of recording my thoughts and was pressured for time. I was not able to do the reflective practice after each and every lesson.

5.5 CONCLUSION AND RECOMMENDATIONS

This study emphasized the use of concrete teaching and learning materials to model fractions using the linear, set, and area models of fraction representation. The learners improved their conceptual understanding of fractions, their abilities to complete fraction activities, and their ability to justify their solutions verbally and in writing. Analysing the data

during this study led me to an additional question. How can activities using concrete teaching and learning materials to represent fractions using multiple models be used to enhance students' problem-solving abilities?

It appears as if Grade 7 learners were prepared to benefit from the use of concrete teaching and learning materials, because their scores in the Fraction test, Mathematics examination and in English language examination were high and acceptable (see Appendix D). Fraction proficiency is vital to success in mathematics in both primary school and high school. The use of concrete teaching and learning materials coupled with numerous teaching strategies in the classroom is one way to provide learners with active, engaging lessons that promote mathematical proficiency.

Teachers should not rush to provide learners with different algorithms to execute operations on fractions, but should place more emphasis on the conceptual understanding of fractions. This study also suggests that the number line comprises a difficulty model for learners to manipulate, so teachers must assist learners to master this model. During the teaching and learning session, I taught learners as a class, completed RNP activities in groups using selected concrete teaching and learning materials, and built in learners' prior knowledge about fractions. During the interviews on sharing chocolates, although learners (both boys and girls) were able to move in between and among the translations, only a few learners were able to reach the written symbol translation. Now I can strongly say that teaching mathematics is less a pumping in of mathematics content and skills (Behaviourism), but more an active participation of both learners and teachers (Constructivism). A teacher can also move from constructivism to modelling, with the aid of RNP activities such as the ones that I implemented on this study. So, I can say that selected concrete teaching and learning materials such as, Fraction strips, Fraction circles, discrete materials and Number line helped in developing proficiency in fraction in Grade 7 learners. The pedagogical practices and strategies that were helpful in developing mathematical proficiency in fractions were hands – on and minds – on RNP activities, use of selected concrete teaching and learning materials, problem solving, learner – centred instruction, group – work and individual – work.

As a result of this study, I plan to continue to incorporate activities that involve the use of selected concrete teaching and learning materials. I plan to provide more opportunities to discuss solving fraction problems in context and how to use the context of the problem to

help determine which model of fraction representation would be most useful. It is my hope that continued use of multiple models of fraction representations will deepen my students' conceptual understanding of fractions and help them to improve their problem-solving skills.

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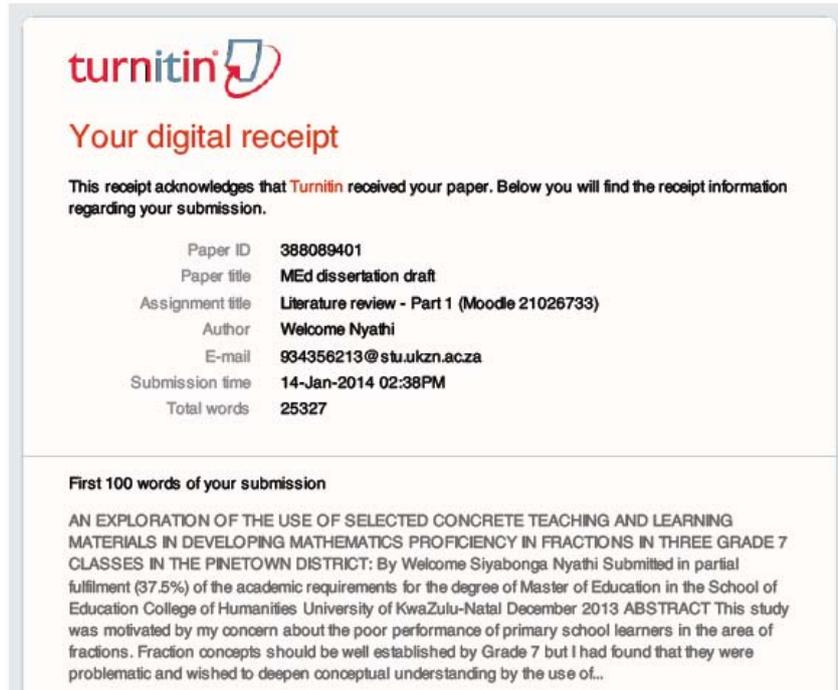
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APPENDICES

APPENDIX A TURNITIN DOCUMENTATION



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AN EXPLORATION OF THE USE OF SELECTED CONCRETE TEACHING AND LEARNING MATERIALS IN DEVELOPING MATHEMATICS PROFICIENCY IN FRACTIONS IN THREE GRADE 7 CLASSES IN THE PINETOWN DISTRICT: By Welcome Siyabonga Nyathi Submitted in partial fulfillment (37.5%) of the academic requirements for the degree of Master of Education in the School of Education College of Humanities University of KwaZulu-Natal December 2013 ABSTRACT This study was motivated by my concern about the poor performance of primary school learners in the area of fractions. Fraction concepts should be well established by Grade 7 but I had found that they were problematic and wished to deepen conceptual understanding by the use of...

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**AN EXPLORATION OF THE USE OF SELECTED
CONCRETE TEACHING AND LEARNING MATERIALS
IN DEVELOPING MATHEMATICS PROFICIENCY IN
FRACTIONS IN THREE GRADE 7 CLASSES IN THE
PINETOWN DISTRICT:**

By

Paper Info	
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Welcome Siyabonga Nyathi

(37.5%) of the degree of

APPENDIX B ETHICAL CLEARANCE DOCUMENTATION



28 October 2009

Mr W S Nyathi
1891 Main Avenue
P O CLERNAVILLE
3620

Dear Mr Nyathi

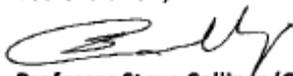
PROTOCOL: An Exploration of the use of multiple visual aids in developing mathematics proficiency in fractions in grade seven, an action research study
ETHICAL APPROVAL NUMBER: HSS/0743/2009: Faculty of Education

In response to your application dated 15 October 2009, Student Number: **934356213** the Humanities & Social Sciences Ethics Committee has considered the abovementioned application and the protocol has been given **FULL APPROVAL**.

PLEASE NOTE: Research data should be securely stored in the school/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully



Professor Steve Collings (Chair)
HUMANITIES & SOCIAL SCIENCES ETHICS COMMITTEE

SC/sn

cc: S D Hobden
cc: Ms R Govender

1891 Main Avenue
P.O. CLERNAVILLE
3602
2009 March 20

Dear Parent / Guardian

Request to work with your child

I am a practicing teacher who is currently furthering his studies at M.Ed. (Mathematics Education) level in the University of KwaZulu-Natal. I have reached a critical point in my studies where I am required to conduct research on any important aspect of mathematics education of my choice. I have chosen to research the use of visual aids in the teaching learning of fraction concept in Grade 7 learners.

In my investigation I need to work with Grade 7 learners, one of whom is your child. I therefore request your permission to work with child, in the company of other Grade 7 learners. You are assured that real names of participants will not be revealed upon the release of findings of the study. You are also assured that findings will not be used for any purposes other than those to do with the objectives of the study.

The work forms part of the normal school curriculum, but you may choose for your child's work not to be included in this study.

In further inquiries, you may contact Dr S.D. Hobden, my supervisor at the university at the following number: 031-260 3435 (office hours).

Yours truly,
W. S. Nyathi

UKZN Student Number: 934356213

I.....**(full names of participant) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research project.**

I understand that I am at liberty to withdraw from the project at any time, should I so desire.

SIGNATURE OF PARTICIPANT

DATE

.....

University of KwaZulu-Natal
Edgewood Campus
Private Bag X 03
ASHWOOD
3605
20 May 2009

The Principal
Zakhele Primary School
P.O. Box 75
CLERNAVILLE
3602

SIR

Letter of Request:

I am a student at the above-mentioned institution, and wish to make a request to conduct a research among learners in your institution. I am a masters' student tasked to research on fractions in Mathematics. My specific research project will focus on the use of visual aids in the teaching – learning of fraction concept in Grade 7 learners.

I will take upon myself to respect local customs and school's image and also promise to give copies of all field reports on request by the school.

In further inquiries, you may contact Dr S.D. Hobden, my supervisor at the university at the following number 031-260 3435 (office hours).

Yours truly,
W.S. Nyathi

UKZN Student Number: 934356213

I ----- **Principal of** -----
----- **does / does not consent, to the above**
in my institution.

SIGNATURE OF PRINCIPAL

DATE

University of KwaZulu-Natal
Edgewood Campus
Private Bag X 03
ASHWOOD
3605
20 May 2009

The School Governing Body
Zakhele Primary School
P.O. Box 75
CLERNAVILLE
3602

Dear Sir / Sirs

Letter of Request:

I am a student at the above-mentioned institution, and wish to make a request to conduct a research among learners in your institution. I am a masters' student tasked to research on fractions in Mathematics. My specific research project will focus on the use of visual aids in the teaching-learning of fraction concept in Grade 7 learners.

I will take upon myself to respect local customs and school's image and also promise to give copies of all reports on request by the school.

In further inquiries, you may contact Dr S.D. Hobden, my supervisor at the university at the following number: 031- 260 3435 (office hours).

Yours truly,
W.S. Nyathi

UKZN Student Number: 934356213

I ----- **Member of the School's Governing Body of**
----- **does / does not consent, to the above in my**
institution.

SIGNATURE OF GOVERNING BODY

DATE

1891 Main Avenue
P.O. CLERNAVILLE
3602
2009 Mashi 20

Mzali

Isicelo sokusebenza nabantwana

Nginguthishela oqhuba izifundo zakhe ze M.Ed. (Mathematics Education) e-University of Kwazulu Natal. Sengifike esigabeni sokuba ngenze ucwaningo ngayinoma iyiphi ingxenye ebalulekile ekufunweni kwe-Mathematics. Ngikhethe ukugxila ekufunweni kwamaqhezu (Fractions) nabantwana baka-grade 7.

Kuloluphenyo ngidinga ukusebenza nabafundi baka-grade 7, omunye wabo okungumntwana wakho. Ngakhoke ngiyacela ungivumele ngisebenze nomntwana wakho, ehlangene nabanye afunda nabo. Ngiyakuqinisekisa ngokuthi asiyukubakhona isidingo sokudalulwa kwamagama abantwana uma sekukhishwa imiphumela, futhi imiphumela angeke isetshenziselwe ezinye izinhloso ngaphandle kwezaloluphenyo. Ukubamba iqhaza komntanakho kuncike othandweni lwakho njengomzali, kanye nakuye umntwana uqobo lwakhe. Umntwana uyovumeleka ukuyeka ukuba yingxenye yalomsebenzi nayinoma yinini uma wena noma yena efisa kube njalo.

Uma kungenzeka ube nemibuzo mayelana naloludaba, uvumelekile ukushayela ucingo uthisha wami khona e-University of KwaZulu Natal, uDr S.D. Hobden, kulenombolo: 031-260 3435 (ngezikhathi zomsebenzi).

Yimina Ozithobayo

U- W.S. Nyathi

UKZN Inombolo Yomfundi: 934356213

Funda bese usayina,

Mina----- ongumzali ka-----

Ngiyavuma ukuthi ngiyayiqonda kahle yonke into ebhalwe ngasenhla futhi ngiyahambisana nayo. Ngiyakuqonda futhi nokuthi umntwana akaphoqekile ukuqhubeka nokuba yingxenye yalomsebenzi, nokuthi angahoxa noma nini uma yena noma mina sifisa kube njalo. Ngiyakuqonda futhi nokuthi amagama angempela ezingane kanye nesikole okube yingxenye yalomsebenzi angeke adalulwe, kepha ayohlale evikelekile ngaso sonke isikhathi.

Signature:-----.

1891 Main Avenue
P.O. CLERNAVILLE
3602
2009 May 26

Dear Colleague

Re: Letter of Consent

I am a M.Ed. student in Mathematics Education at the University of KwaZulu-Natal, and my course presently requires me to research in an area of importance to the teaching and learning of mathematics. My area of interest is the use of visual aids in the teaching-learning of fraction concepts in Grade 7, with a view to enhance understanding of conceptual meaning involved. This requires me to work with experienced educators like you, people with the necessary expertise in the subject.

The research project requires me amongst others to:

1. To teach a set of mathematics lessons on the fractions concept using concrete teaching and learning materials.
2. Administer Rational Number Project (RNP) activities during the study.
3. To interview learners on the use of concrete teaching and learning materials.
4. To test learners on fractions.
5. To use Mathematics and English language examination marks on the study.

Your co-operation in respect of the five areas mentioned above is invaluable and will be highly appreciated.

Your participation in this project is entirely voluntary, and should you at any stage wish to withdraw, you will be free to do so. You are assured of complete confidentiality of your identity as a participant in this project. No real names, either of persons / institutions, will be used in the write-up of the findings of this study. Also findings of this study will be used for no purposes other than those of the study. Should you be interested in the findings of this study, theses will be made available to you through the necessary arrangements.

In the event of you having any questions, you are free to contact my supervisor Dr S.D. Hobden, at 031-260 3435 (office hours).

Yours truly,
W.S. Nyathi

UKZN Student Number: 934356213

Please read and sign

I -----, fully understand the conditions of my participation in this project. I also understand that this participation is voluntary, and can be terminated as and when I think necessary. I also understand that no real names will be used in the write-up of this study.

Signature:-----

APPENDIX C INTERVIEW SAMPLE

STRATIFIED SAMPLE:

PSEUDONYM	GROUP	GENDER	AGE
Vusi	7C	Male	15 years
Pinky	7C	Female	13 years
Thandi	7C	Female	13 years
Mxolisi	7B	Male	12 years
Ntombi	7A	Female	16 years
Percy	7B	Male	15 years
Gugu	7C	Female	16 years
Themba	7B	Male	15 years
Thobile	7A	Female	14 years
Tom	7A	Male	13 years
Bridget	7B	Female	14 years
Winston	7B	Male	13 years

APPENDIX D STATISTICAL ANALYSIS

ANOVA Fraction test scores of the three classes

Descriptives

Fractions test								
	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Grade 7A	50	53.30	10.576	1.496	50.29	56.31	15	75
Grade 7B	49	55.20	12.074	1.725	51.74	58.67	40	90
Grade 7C	51	62.35	13.614	1.906	58.52	66.18	40	90
Total	150	57.00	12.705	1.037	54.95	59.05	15	90

ANOVA

Fractions test					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	2303.894	2	1151.947	7.787	.001
Within Groups	21746.106	147	147.933		
Total	24050.000	149			

Multiple Comparisons

Dependent Variable: Fractions test

Scheffe

(I) Class	(J) Class	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Grade 7A	Grade 7A					
	Grade 7B	-1.904	2.445	.739	-7.95	4.14
	Grade 7C	-9.053*	2.421	.001	-15.04	-3.07
Grade 7B	Grade 7A	1.904	2.445	.739	-4.14	7.95
	Grade 7B					
	Grade 7C	-7.149*	2.433	.015	-13.17	-1.13
Grade 7C	Grade 7A	9.053*	2.421	.001	3.07	15.04
	Grade 7B	7.149*	2.433	.015	1.13	13.17
	Grade 7C					

*. The mean difference is significant at the .05 level.

ANOVA fraction test scores of boys and girls

Descriptives

Fractions test										
	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum		
					Lower Bound	Upper Bound				
Boy	68	55.88	10.472	1.270	53.35	58.42	40	85		
Girl	82	57.93	14.293	1.578	54.79	61.07	15	90		
Total	150	57.00	12.705	1.037	54.95	59.05	15	90		

ANOVA

Fractions test						
	Sum of Squares	df	Mean Square	F	Sig.	
Between Groups	155.380	1	155.380	.962	.328	
Within Groups	23894.620	148	161.450			
Total	24050.000	149				

ANOVA fraction test scores of age groups

Descriptives

Fractions test										
	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum		
					Lower Bound	Upper Bound				
11	1	70.00	70	70		
12	26	58.27	13.412	2.630	52.85	63.69	40	90		
13	53	58.68	12.678	1.741	55.18	62.17	40	90		
14	37	56.76	11.798	1.940	52.82	60.69	15	80		
15	19	54.21	14.459	3.317	47.24	61.18	20	80		
16	12	52.92	11.373	3.283	45.69	60.14	40	85		
17	2	45.00	.000	.000	45.00	45.00	45	45		
Total	150	57.00	12.705	1.037	54.95	59.05	15	90		

ANOVA

Fractions test						
	Sum of Squares	df	Mean Square	F	Sig.	
Between Groups	998.452	6	166.409	1.032	.407	
Within Groups	23051.548	143	161.200			
Total	24050.000	149				

ANOVA – Class groups in Gr 7 mathematics exam

Descriptives

Grade 7 Maths								
	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Grade 7A	50	54.20	10.821	1.530	51.12	57.28	40	82
Grade 7B	49	50.55	7.948	1.135	48.27	52.83	42	74
Grade 7C	51	61.53	14.234	1.993	57.53	65.53	40	85
Total	150	55.50	12.162	.993	53.54	57.46	40	85

ANOVA

Grade 7 Maths					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	3138.672	2	1569.336	12.205	.000
Within Groups	18900.828	147	128.577		
Total	22039.500	149			

Multiple Comparisons

Dependent Variable: Grade 7 Maths

Scheffe

(I) Class	(J) Class	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Grade 7A	Grade 7A					
	Grade 7B	3.649	2.279	.281	-1.99	9.29
	Grade 7C	-7.329*	2.257	.006	-12.91	-1.75
Grade 7B	Grade 7A	-3.649	2.279	.281	-9.29	1.99
	Grade 7B					
	Grade 7C	-10.978*	2.268	.000	-16.59	-5.37
Grade 7C	Grade 7A	7.329*	2.257	.006	1.75	12.91
	Grade 7B	10.978*	2.268	.000	5.37	16.59
	Grade 7C					

*. The mean difference is significant at the .05 level.

ANOVA Class groups in Grade 7 English examination

Descriptives

Grade 7 English								
	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Grade 7A	50	54.92	8.253	1.167	52.57	57.27	42	72
Grade 7B	49	51.96	8.359	1.194	49.56	54.36	40	74
Grade 7C	51	56.00	11.586	1.622	52.74	59.26	40	82
Total	150	54.32	9.637	.787	52.77	55.87	40	82

ANOVA

Grade 7 English						
	Sum of Squares	df	Mean Square	F	Sig.	
Between Groups	435.042	2	217.521	2.386	.096	
Within Groups	13403.598	147	91.181			
Total	13838.640	149				